Exercise 1:

1. Show that each IVP

\[ y' = y^2, \quad y(x_0) = y_0 \]

has a unique solution on an appropriate interval centered at \( x_0 \).

2. Find a solution of the IVP in (1) with \( y(0) = 1 \), and determine the largest interval \( I \) of definition for the solution.

3. Solve the IVP

\[ y' = y^2, \quad y(0) = 0 \]

Solution:
Exercise 2: Solve the following DE:

\[ xy' = y^2 - y \]

and show that it has a singular solution.

Find the solution that passes through the indicated points:

(a) \((1,1)\)
(b) \((0,0)\)

Solution:
Exercise 3: Solve the following IVP and find the interval of validity of the solution:

\[(x^2-1) y' + 2y = (x+1)^2, \quad y(2) = 5\]

Solution: