Exercise 1:

Let $P$ and $Q$ be statements. Which of the following implies that $P \lor Q$ is false?

(a) $(\sim P) \lor (\sim Q)$ is false
(b) $(\sim P) \lor Q$ is true
(c) $(\sim P) \land (\sim Q)$ is true.
(d) $Q \Rightarrow P$ is true.
(e) $P \land Q$ is false.
Exercise 2:

For statements $P$ and $Q$, show that $(P \land (P \Rightarrow Q)) \Rightarrow Q$ is a tautology. Then state $(P \land (P \Rightarrow Q)) \Rightarrow Q$ in words. (This is an important logical argument form, called modus ponens.)
Exercise 3

For statements $P$, $Q$ and $R$, show that $((P \Rightarrow Q) \land (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$ is a tautology. Then state this compound statement in words. (This is another important logical argument form, called syllogism.)
Exercise 4:

Let $R$ and $S$ be compound statements involving the same component statements. If $R$ is a tautology and $S$ is a contradiction, then what can be said of the following?

(a) $R \lor S$  
(b) $R \land S$  
(c) $R \Rightarrow S$  
(d) $S \Rightarrow R$. 
Exercise 5:

Using algebraic properties of logical equivalence;

(a) For statements $P$, $Q$ and $R$, show that

\[(P \land Q) \Rightarrow R \equiv ((P \land (\sim R)) \Rightarrow (\sim Q))\].

(b) For statements $P$, $Q$ and $R$, show that

\[(P \land Q) \Rightarrow R \equiv ((Q \land (\sim R)) \Rightarrow (\sim P))\].
Exercise 6:

Assign propositional variables to come up with the general argument, and then show that the argument is valid. What rules of inference are used?

Argument:
If I drink coffee, then I will get a lot of work done.
If I don’t drink coffee, then I am sleepy.
If I am sleepy, then I am grumpy.
Therefore if I don’t get a lot of work done, then I am grumpy.
Exercise 7:
An island contains two types of people, knights and knaves. Knights always tell the truth, and knaves always lie. You go to the island, and two people approach you.
Person A says: B is a knight.
Person B says: A and I are of opposite type.

What type are A and B?