Math 440-142 Major Exam I

(1) If all normal to a regular parametrised space curve pass through a fixed point, show that its torsion is 0 and its curvature is constant.

(You may use this as a starting point: Parametrize the curve α by arc length s. Let $n(s)$ be the normal field along α. You are given that $\alpha(s) + \lambda(s)n(s) = constant$. Differentiate this and use Serret-Fremet formulae)

(2)(a)Find a parametrization for the circular cone obtained by rotating the line $y = z$ in the $(y, z) - $ plane about the $z - axis$.
(b) Find a basis for the tangent space at any point of the cone- including the origin.
(c) Find a parametrization for a general cone $\frac{(x-a)^2}{l^2} + \frac{(y-b)^2}{m^2} - \frac{(z-c)^2}{n^2} = 0$

(3) Find the first fundamental form for a surface of revolution and use it to find the coefficients of the first fundamental form for a torus.
Also, set up a double integral for the area of the torus.
(The circle in $(y, z) -$ plane centered at $(0, R, 0)$ of radius $r < R$ is rotated about the $z - axis$. The surface of revolution you get is a torus).

(4) (a)Define a smooth surface. Show a sphere of radius R is a smooth surface

(b). Show that any point on the sphere lies on a great circle joining two fixed anti-podal points
(c) Deduce that any two points on a sphere lie on some great circle.