Math 440-142 Final Exam

Name:____________________ id#:__________________

Write answers so that they are readable. No credit for answers that I cannot read.

Q1) A plane unit speed curve γ is such that $\gamma(s)$ makes a constant angle θ with its tangent vector $t(s)$. Show that the curvature function is given by $k(s) = \frac{-sin\theta}{scos\theta+a}$

Q2) (a). Give a precise definition of a smooth surface in $\mathbb{R}^3$.

(b) A curve in the $(x,y)-$ plane is rotated about the $x-$ axis. Write down a parametrization for the surface and determine conditions for this parametrization to be regular.

(c) If you rotate a circle of radius 1 in the plane about the $x-$ axis and use the parametrization in (b), at what points – if any- is this parametrization irregular.

Q3) Let $F(x,y,z)$ and $G(x,y,z)$ be a differentiable functions. Prove that if a local Maximum or minimum of F on a level set of G is achieved at a point p where gradient of G is non-zero, then necessarily $grad(F)(p) = \lambda grad(G)(p)$ for some constant λ.

Q4) Prove that if the maximum and minimal values of the 2^{nd} fundamental form are $\kappa_{1,}\, \kappa_2$ and they are achieved in perpendicular orthonormal directions and $v = (cos\theta)\, e + (sin\theta)\, f$, then $\int_0^{2\pi} \Pi_2\,(v)d\theta = 2\pi H$, where H is the mean curvature.

Q5) (a). Find the Gaussian and mean curvature of a surface of revolution obtained from a curve parametrized by arc length.

(b) Solve the differential equation for the Gaussian curvature from (a) to find all such surfaces whose Gaussian curvature is 1.