King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 531 (Real Analysis) **Term 142**

Exam 1 : March 09, 2015 **Time allowed: 2hrs**

Question Number	Marks	Maximum Points
1		2
2		2
3		2
4		2
5		2
6		2
7		2
8		2
9		4
Total		20

(Q1) Define Lebesgue outer measure m^* of a set A of real numbers.
Prove that $m^*(A + y) = m^*(A)$ where y is any real numbers.

(Q2) If F is a measurable set and $m^*(F \triangle G) = 0$, then show that G is a measurable set.

(Q3) If E_1 and E_2 are measurable sets, then prove that
$m(E_1 \cup E_2) = m(E_1) + m(E_2) - m(E_1 \cap E_2)$ where m stands for the Lebesgue measure.

(Q4) Let m denote the Lebesgue measure and $\{E_n\}$ be a decreasing sequence of measurable sets with $m(E_1) < \infty$. Then prove that $m\left(\bigcap_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} m(E_n)$.
Show by means of an example that the condition $m(E_1) < \infty$ is necessary for the conclusion.

(Q5) Define Cantor set C. Prove that C is uncountable and $m(C) = 0$.

(Q6) Outline the procedure to demonstrate the existence of a non-Lebesgne measurable set. Hence show by an example that if $|f|$ is measurable function, then f may not be measurable function.

(Q7) Let $\{f_n\}$ be a sequence of extended real-valued measurable functions with the same domain D. Prove that $\overline{\lim} f_n$ and $\{x \in D : f_1(x) > f_2(x)\}\}$ are measurable.

(Q8) Let $f : \mathbb{R} \to \mathbb{R}$ be a Lebesgue measurable function and $g : \mathbb{R} \to \mathbb{R}$ a Borel measurable function. Prove that gof is a Lebesgue measurable function.

(Q9) Prove or disprove as briefly as possible:

(a) Every Lebesgue measurable set is a Borel set.

(b) Composition of two Lebesgue measurable functions is a Lebesgue measurable function.