(1) Let $\Omega = (0,1)$ and $f(x) = x$ $g(x) = \cos(x)$. Verify Cauchy-Schwarz inequality.

(2) [Problem A.5 Page 240]

(3) Give vibrational formulation for the problem:

$$-u'' = f$$

with $u'(0) = u'(1) = 0$

and explain why this problem is not well-posed.

(4) Modify the Matlab code FEM_1D.m to solve the following problem

$$-u'' + b(x)u' + c(x)u = f(x)$$

with $u(0) = u(1) = 0$

where $b(x) = -x^3$ $c(x) = x^2$ $f(x) = (16\pi^2 + x^2) \sin(4\pi x) - 4\pi^3 \cos(4\pi x)$

Then approximate the value of $u$ at $x = 0.5$ and plot the solution.