(1) [Problem 5.7 Page 74]

(2) Consider the Neumann problem

\[- \nabla \cdot (a \nabla u) + cu = f, \text{ in } \Omega\]

\[\frac{\partial u}{\partial n} = 0 \text{ on } \partial \Omega\]

where \(a(x) \geq a_0 > 0, c(x) \geq c_0 > 0\) Formulate a finite element problem and then prove error estimate.

(3) Consider the 9-point approximation to the laplacian

\[- \Delta_{9 \text{pt}}^\theta = - \left( \theta \Delta_h + (1 - \theta) \Delta_{2h} \right)\]

a) Find the value of \(\theta\) which makes \(- \Delta_{9 \text{pt}}^\theta O(h^4)\) accurate.

b) Represent this as a stencil.

(4) Consider the elliptic equation

\[- (au_{xx} + 2bu_{xy} + cu_{yy}) = f, \text{ in } \Omega\]

\[u = g \text{ in } \partial \Omega\]

\[\Omega = (0,1) \times (0,1) \quad b^2 < ac\]

Discretize the mixed derivative by \(\frac{\partial^2}{\partial x \partial y} \approx \partial_1 \partial_2 + \overline{\partial_1} \overline{\partial_2}\)

a) What is the order of accuracy is attained

b) Under what conditions is this of positive type?

(5) In the proof of the Theorem 4.4 page 47, show the following

a) \(W_i \geq 0\)

b) \(- \Delta_i W_i = 4\)

c) \(W_i \leq 1/2\)

d) \(V_0^\perp \leq |U|_r\)