Q1] [10 points] A test consists of 10 multiple choice questions, each with five possible answers, one of which is correct. If a student randomly guesses,

1. (4 points) What is the probability that the student will get at least two correct answers?

2. (4 points) What is the probability of getting the number of correct answers greater than the number of incorrect answers?

3. (2 points) If $X$ represents the number of correct answers, find $E(X^2)$. 
Q2]...[10 points] Let $X$ have a geometric distribution.

1. (5 points) Show that $P(X > n) = (1 - p)^n$.

2. (5 points) Show that $X$ has the following memoryless property:

\[ P(X > k + j | X > k) = P(X > j) \]

where $k$ and $j$ are nonnegative integers.
Q3]...[10 points] Roll a fair die twice. Let $X_1$ be the number of times that face 1 shows, and let $X_2 = \left\lfloor \frac{\text{sum of faces}}{4} \right\rfloor$ where $[x]$ denotes the integer part of $x$.

1. (5 points) Construct the joint probability mass function of $X_1$ and $X_2$.

2. (3 points) Are $X_1$ and $X_2$ independent?

3. (2 points) Calculate the expected value of $X_1$. 
Q4]...[10 points] Let \((X, Y)\) have a joint density given by

\[
f(x, y) = \begin{cases} 
\lambda_1 \lambda_2 e^{-\lambda_1 x - \lambda_2 y} & \text{for } x, y \geq 0, \\
0 & \text{otherwise}
\end{cases},
\]

1. (5 points) Prove that

\[
P(X < Y) = 1 - \frac{E(X)}{E(X) + E(Y)}.
\]

2. (5 points) Find the probability density function for \(\frac{X}{Y}\).
Q5]...[10 points] Suppose that $X$ has gamma distribution with $(\alpha = 2, \lambda)$, and that $Y$ has uniform distribution in $(0, x)$ given $X = x$.

1. (5 points) Find the joint density of $X$ and $Y$.

2. (5 points) Compute the marginal density of $Y$. 

GOOD LUCK