Q.No.1: (5+5+5+5=20 points)
(a) If \( k \) trials conducted are of Bernoullian type following binomial distribution, find the maximum likelihood estimate of \( p \).

(b) Find the maximum likelihood estimate of the parameter \( \theta \) of the following distribution,
\[
f(x; \theta) = \frac{1}{2} e^{-|x-\theta|}
\]
\(-\infty < x < \infty \) and \(-\infty < \theta < \infty\).

(c) Estimate the parameter \( \beta \) of the distribution \( f(x; \beta) = \beta e^{-\beta x} \) for \( 0 \leq x \leq \infty \), by the method of moments.

(d) Find Bayes estimator of the single parameter \( \theta \) of the Poisson distribution \( f(x; \theta) = e^{-\theta} \theta^x / x! \) for \( x = 0, 1, 2, \ldots, n \) when it is known that the prior distribution of \( \theta \) is gamma distribution \( g(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta \theta} \) for \( 0 \leq \theta \leq \infty \).

Q.No.2: (7 points) Consider a distribution having a pmf of the form \( f(x; \theta) = \theta^x (1 - \theta)^{1-x} \) for \( x = 0, 1 \). Let \( H_0: \theta = \frac{1}{20} \) against \( H_1: \theta > \frac{1}{20} \). Use the central limit theorem to determine the sample size \( n \) of a random sample so that the uniformly most powerful test of \( H_0 \) against \( H_1 \) has a power function \( \gamma(\theta) \), with approximately \( \gamma \left( \frac{1}{20} \right) = 0.05 \) and \( \gamma \left( \frac{1}{10} \right) = 0.90 \).

Q.No.3: (8 points) Let \( X_1, X_2, \ldots, X_n \) denote a random sample from a gamma distribution \( f(x; \theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \) for \( x > 0 \), \( \mu = \alpha/\beta \) and \( \sigma^2 = \alpha/\beta^2 \) with \( \alpha = 2 \) and \( \beta = \theta \). Let \( H_0: \theta = 1 \) against \( H_1: \theta > 1 \). Show that the likelihood ratio test leads to the same critical region as that given by the Neyman-Pearson lemma. Also find the value of \( k \) using \( \alpha = 0.05 \).

With the Best Wishes