Q1. Indicate the correct choice. (10 points)

1. Statistical process control is:
   a. A technique for finding the best settings on machines or equipment
   b. A method of ensuring consistent levels of product quality by monitoring the production process.
   c. A way to identify and eliminate potential failure modes in an operation.
   d. A means of ensuring that the “voice of the customer” is considered at every step of design and production.

2. All of the output from a single source is called a:
   a. Process
   b. Population
   c. Subgroup
   d. Control Chart

3. An individual item or product that’s selected at random for inspection or evaluation is called a:
   a. Subgroup
   b. Sample
   c. Population
   d. Predictor

4. A group of items or products that are selected in sequence from a single source and used to predict the quality of all the items from that source is called a:
   a. Sample
   b. Subgroup
   c. Population
   d. Product series

5. A solid horizontal line across a control chart represents the:
   a. Average process speed.
   b. Average of all the subgroup values.
   c. Target value for the population.
   d. Largest acceptable value.

6. The upper and lower control limits on a control chart are:
   a. Expected variations among individual products.
   b. Calculated from actual measurement data.
   c. Determined during product design
   d. The same as specification limits.
7. An angle of taper is expressed as 42.75°. What type of data is this?

a. Categorical data  
b. Attribute data  
c. Variables data  
d. None

8. Which of the control charts listed would be most appropriate for the type of data collected on the width of a flange, measured and expressed to 0.0001 inches

a. np chart  
b. p chart  
c. X-bar/R chart  
d. c chart

9. On a control chart, when points remain inside the control limits and vary under a pattern on either side of the central line, it indicates that:

a. The process is operating as expected.  
b. The process is working under natural variations  
c. A problem may exist in the process and should be investigated.  
d. Both a and b

10. A way of displaying possible causes of a problem in a shape that resembles a fishbone is known as:

a. Check sheet  
b. Pareto Chart  
c. Cause-and-Effect Diagram  
d. Pareto Diagram

11. A type of bar chart in which items are ranked highest to lowest according to their frequency and/or cost.

a. Cause-and-Effect Diagram  
b. Frequency Table  
c. Pareto Diagram  
d. Check Sheet

12. Which one is true for rational subgrouping:

a. It minimizes the chance of variability due to assignable causes within a sample,  
b. It maximizes the chance of variability between samples if assignable causes are present.  
c. Both a and b  
d. It is of no use for process monitoring
Q2. Define the following terms. Note that definitions must be very brief and should not exceed the space provided. (10 points)

1. **Natural Variations:**

2. **Statistical Process Control**

3. **False Alarm Rate**

4. **Transient Shifts**

5. **Pivotal Quantity**
6. Median Run Length

7. Inertia

8. Retrospective Analysis

9. Process Capability

10. Average Time to Signal
Q3. (10 points)

(a). Define the capability indices $C_{pm}$ and $C_{pm}^*$. Using a bijective transformation of $C_{pm}^*$ define $C_{pp}$ index and show the derivation of its partitioning in the form of inaccuracy and imprecision indices separately.
(b). Define EWMA for individual observations and derive its variance.
Q.4. Let X be a continuous random variable (quality characteristic of interest in a process) with mean $\mu$ and standard deviation $\sigma$ (i.e. variance $\sigma^2$) and is normally distributed, i.e. $X \sim N(\mu, \sigma^2)$. Let $x_1, x_2, ..., x_n$ be a random sample of size n from the process. Using this information, provide computations for the following process scenarios.

Using 25 phase I in-control samples of size n=2, we got $\bar{x} = 0.0296$, $\bar{R} = 1.3167$, $\bar{S} = 0.9311$. For $\bar{X}$, R and S charts, provide the following.

i). 3-sigma control limits:

ii). False alarm rates for all the limits computed in part (i) and their corresponding ARL0 values. Also interpret your results.
iii). Probability Limits at false alarm rate of 0.005.

iv). If the process variance remains stable at $\sigma_0^2$ but mean shifts to new level $\mu_i = \mu_0 + \delta \sigma_0$. Using the limits computed in part (iii) for $\bar{X}$ chart, evaluate the power at $\delta = -0.25$ and 0.75 and find their corresponding ARL1 values. Also interpret your results.
v). If the process variance shifts to a new level \( \sigma_i^2 = \omega \sigma_0^2 \). Using the limits computed in part (iii) for S chart, evaluate the power at \( \omega = 1.00, 1.25 \) and \( 1.75 \) and find their corresponding ARL1 values. Also interpret your results.

vi). If you have to repeat part (v) for R chart. What kind of results do you expect? Justify your answer with arguments (no computations required).
Q.5. Let \( \mathbf{X} \) be a continuous random vector (of correlated quality characteristics of interest in a process) with mean vector \( \mathbf{\mu} \) and variance covariance matrix \( \mathbf{\Sigma} \) and is normally distributed, i.e. \( \mathbf{X} \sim \mathcal{N}_p(\mathbf{\mu}, \mathbf{\Sigma}) \). Let \( \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n \) be a random sample of size \( n \) from the process. Consider \( p=2, n=4 \), and the case of known parameters as specified below:

\[
\mathbf{\mu}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{\Sigma}_0 = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}
\]

Using this information:

a). Provide the following for Chi-squared control chart:

i). Plotting statistic:

ii). Control limits at the false alarm rate of 0.005:

b). Provide the following for the generalized variance chart based on \( |S|^{1/2} \):

i). Control limits (two-sided) at the false alarm rate of 0.005
ii). If the process variance covariance matrix shifts to a new level defined as: \( \Sigma_1^{1/2} = \delta \Sigma_0^{1/2} \).

Using the limits computed in part (i), evaluate the power at \( \delta = 0.5, 1.00, 1.5 \) and find their corresponding ARL1 values.

Q.6. For a location control chart, following is the table of ARL values against their respective shifts quantified by \( \delta \). Using this information, compute the value of EQL value and interpret it.

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>0.00</th>
<th>0.20</th>
<th>0.40</th>
<th>0.60</th>
<th>0.80</th>
<th>1.00</th>
<th>1.20</th>
<th>1.40</th>
<th>1.60</th>
<th>1.80</th>
<th>2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARL</td>
<td>375.38</td>
<td>180.57</td>
<td>57.33</td>
<td>20.97</td>
<td>9.03</td>
<td>4.57</td>
<td>2.70</td>
<td>1.83</td>
<td>1.40</td>
<td>1.18</td>
<td>1.08</td>
</tr>
</tbody>
</table>