King FahdUniversity of Petroleum and Minerals
Department of Mathematics and Statistics

Math 101
Final Exam
Term 143
Tuesday 11/8/2015
Net Time Allowed: 180 minutes

MASTER VERSION

1. If $A = \lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$ and $B = \lim_{x\to 0} \sqrt{7+\sec^2 x}$, then $6A + \sqrt{2}B =$

$A \overset{L.R}{=} \lim_{x\to 9} \dfrac{\frac{1}{2\sqrt{x}}}{1} = \dfrac{1}{2\sqrt{9}} = \dfrac{1}{6}$

$B = \lim_{x\to 0} \sqrt{7+\sec^2 x}$

$= \sqrt{7+1} = \sqrt{8}$

$6A + \sqrt{2}B = 6\left(\frac{1}{6}\right) + \sqrt{2}\sqrt{8}$

$= 1 + \sqrt{16}$

$= 1 + 4$

$= 5$

(a) 5

(b) $2\sqrt{2}$

(c) $1 + 2\sqrt{2}$

(d) $1 + \sqrt{2}$

(e) 10

2. The value of $\delta (\delta > 0)$ such that for all x satisfying $0 < |x - 10| < \delta$, the inequality $|\sqrt{14-x} - 2| < 1$ holds, equals to

$0 < |x-10| < \delta$

$-\delta < x-10 < \delta, \; x \neq 10$

$10-\delta < x < 10+\delta, \; x \neq 10$

$|\sqrt{14-x} - 2| < 1$

$-1 < \sqrt{14-x} - 2 < 1$

$1 < \sqrt{14-x} < 3$

$1 < 14 - x < 9$

$-13 < -x < -5$

$5 < x < 13$

Thus:

$10 - \delta = 5 \Rightarrow \delta_1 = 5$

$10 + \delta = 13 \Rightarrow \delta_2 = 3$

$\therefore \; \delta = \min\{\delta_1, \delta_2\} = 3$

(or any smaller positive number)

(a) 3

(b) 5

(c) 13

(d) 10

(e) 9

3. If $\lim_{x\to 0} \dfrac{\sin 4x}{kx - x} = 2$, k is constant, then $k =$

(a) 3

(b) $\dfrac{3}{2}$

(c) 5

(d) 2

(e) $\dfrac{4}{5}$

$\lim_{x\to 0} \dfrac{\sin 4x}{kx - x} = \lim_{x\to 0} \dfrac{\sin 4x}{x(k-1)}$

$= \dfrac{1}{k-1} \cdot \lim_{x\to 0} \dfrac{\sin 4x}{4x} \cdot 4$

$= \dfrac{1}{k-1} \cdot 4 = 2$

$2k - 2 = 4$

$2k = 6 \to k = 3$

4. The values of a and b for which the function

$$f(x) = \begin{cases} a & , \quad x \le 1 \\ \ln(x^4 - 1) - \ln(x - 1) & , \quad 1 < x \le 3 \\ \ln b & , \quad 3 < x \end{cases}$$

is continuous everywhere are:

(a) $a = 2\ln 2,\ b = 40$

(b) $a = \ln 2,\ b = \ln 40$

(c) $a = \ln 2,\ b = 20$

(d) $a = \ln 2,\ b = 10$

(e) $a = 20,\ b = \ln 3$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$ (bcs. conts.)

$a = \lim_{x\to 1^+} \ln\left(\dfrac{x^4-1}{x-1}\right)$

$= \lim_{x\to 1^+} \ln\left(\dfrac{(x^2+1)(x+1)}{1}\right)$

$= \ln 4 = \ln 2^2 = 2\ln 2$

$\therefore \boxed{a = 2\ln 2}$

Also: $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

$\ln 40 = \ln b$

$\Rightarrow \boxed{b = 40}$

5. If $y^2 + x = 3xy$ then $\dfrac{dy}{dx}$ when $y = 1$ is equal to:

(a) 4

(b) $\dfrac{1}{4}$

(c) $\dfrac{8}{7}$

(d) $\dfrac{4}{7}$

(e) $\dfrac{8}{9}$

$2yy' + 1 = 3(xy' + y \cdot 1)$

$2yy' - 3xy' = 3y - 1$

$y'(2y - 3x) = 3y - 1$

$\therefore \quad y' = \dfrac{3y-1}{2y-3x}$

when $y = 1 \Rightarrow 1^2 + x = 3x(1)$

$2x = 1 \rightarrow x = \frac{1}{2}$

$\therefore \left.\dfrac{dy}{dx}\right|_{y=1} = \dfrac{3(1)-1}{2(1) - 3(\frac{1}{2})}$

$= \dfrac{2}{1/2} = 4.$

6. If $y = \sqrt[4]{\dfrac{x^2+1}{1-x^2}}$, then which one of the following statements is **TRUE**?

(a) $y'(1 - x^4) = xy$

(b) $y'(x^4 - 1) = xy$

(c) $y'(1 - x^4) = xy(1 + x^2)$

(d) $y'(1 + x^2) = xy$

(e) $xy' = y(1 - x^4)$

$y = \sqrt[4]{\dfrac{x^2+1}{1-x^2}}$

$\ln y = \frac{1}{4}\left\{ \ln(x^2+1) - \ln(1-x^2) \right\}$

$\dfrac{y'}{y} = \dfrac{1}{4}\left\{ \dfrac{2x}{x^2+1} - \dfrac{-2x}{1-x^2} \right\}$

$= \dfrac{1}{4}\left\{ \dfrac{2x - 2x^3 + 2x^3 + 2x}{(1+x^2)(1-x^2)} \right\}$

$= \dfrac{1}{4}\left\{ \dfrac{4x}{1-x^4} \right\}$

$\therefore \quad \dfrac{y'}{y} = \dfrac{x}{1-x^4}$

$\Rightarrow y'(1-x^4) = xy$

7. A particle is moving along the curve whose equation is $\dfrac{xy^3}{1+y^2} = \dfrac{8}{5}$. If the $x-$ coordinate is increasing at rate of 7 units/s when the particle is at the point $(1, 2)$. At what rate is the y-coordinate of the point changing at this instant?

$5xy^3 = 8 + 8y^2$

$5\left(x \cdot 3y^2 \frac{dy}{dt} + y^3 \cdot \frac{dx}{dt}\right) = 16y \frac{dy}{dt}$

$15xy^2 \frac{dy}{dt} - 16y \frac{dy}{dt} = -5y^3 \frac{dx}{dt}$

At $(1,2)$: we have.

$15(1)(2)^2 \frac{dy}{dt} - 16(2) \frac{dy}{dt} = -5(2)^3(7)$

$28 \frac{dy}{dt} = -280$

$\frac{dy}{dt} = \frac{-280}{28}$

$= -10$ units/s

(a) -10 units /s

(b) -40 units /s

(c) $\dfrac{-10}{7}$ units /s

(d) $\dfrac{10}{7}$ units /s

(e) 10 units /s

8. Given a function $f(x) = x + \cos x$ and an interval $\left[0, \frac{\pi}{2}\right]$. The Mean Value Theorem tells us that there is a number c between 0 and $\frac{\pi}{2}$ such that:

$f'(x) = 1 - \sin x$

$f'(c) = \dfrac{f(\frac{\pi}{2}) - f(0)}{\frac{\pi}{2} - 0}$

$= \dfrac{\frac{\pi}{2} - 1}{\frac{\pi}{2}}$

$= \frac{2}{\pi}\left(\frac{\pi}{2} - 1\right)$

$= 1 - \frac{2}{\pi}$.

(a) $f'(c) = 1 - \dfrac{2}{\pi}$

(b) $f'(c) = 1$

(c) $f'(c) = \dfrac{2}{\pi}$

(d) $f'(c) = \dfrac{1}{1 - \frac{2}{\pi}}$

(e) $f'(c) = 0$

9. If $\dfrac{d^2y}{dx^2} = 2 - 6x$; $y'(0) = 4$, $y(0) = 1$, then $y(1) =$

(a) 5

(b) 0

(c) 4

(d) 1

(e) 9

$\frac{dy}{dx} = y' = \int (2-6x)\,dx$

$= 2x - 3x^2 + C$

$y'(0) = 0 - 0 + C = 4 \rightarrow \boxed{C=4}$

$\therefore\ y' = 2x - 3x^2 + 4$

$y = \int (2x - 3x^2 + 4)\,dx$

$= x^2 - x^3 + 4x + C_1$

but $y(0) = 0 - 0 + 0 + C_1 = 1 \rightarrow C_1 = 1$

$\therefore\ y = x^2 - x^3 + 4x + 1$

so $y(1) = 1 - 1 + 4 + 1 = 5$.

10. Estimating the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles of equal widths and right end points, we get:

(a) 8

(b) 4

(c) $\dfrac{23}{4}$

(d) 6

(e) 3

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{3} = 1$

Subintervals: $[-1,0]$, $[0,1]$, $[1,2]$

$A \simeq 1\,(f(0) + f(1) + f(2))$

$= 1\,(1 + 2 + 5)$

$\simeq 8$

11. $\int \left(\frac{x+1}{x}\right) dx =$

(a) $x + \ln|x| + c$

(b) $\frac{x^2}{2} + \ln x + c$

(c) $x^2 + \ln|x| + c$

(d) $1 + \ln|x| + c$

(e) $\ln x + c$

$\int \left(\frac{x+1}{x}\right) dx = \int \left(\frac{x}{x} + \frac{1}{x}\right) dx$

$= \int \left(1 + \frac{1}{x}\right) dx$

$= x + \ln|x| + C.$

12. To find one root of the function $f(x) = x^3 + ax + b$ we use Newton's method. When $x_0 = 0$, we have $x_1 = 2$, and when $x_0 = 1$ we have $x_1 = \frac{6}{5}$. Then $a - b$ is:

(a) 6

(b) -2

(c) 2

(d) $\frac{-48}{11}$

(e) -6

$f'(x) = 3x^2 + a$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = x_0 - \frac{x_0^3 + ax_0 + b}{3x_0^2 + a}$

When $x_0 = 0 \Rightarrow 2 = 0 - \frac{0 + 0 + b}{0 + a}$

$\Rightarrow \boxed{b = -2a}$

when $x_0 = 1 \Rightarrow \frac{6}{5} = 1 - \frac{1 + a + b}{3 + a}$

$\frac{1}{5} = -\frac{1 + a - 2a}{3 + a}$

$\frac{1}{5} = \frac{-1 + a}{3 + a}$

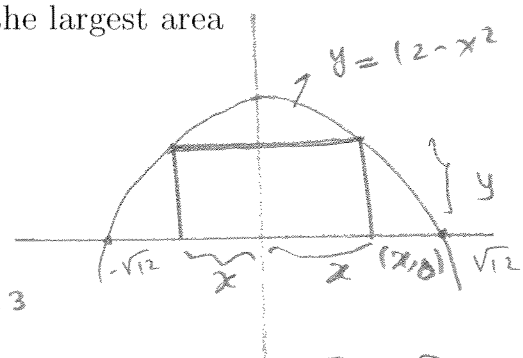
$5a - 5 = 3 + a$

$4a = 8 \rightarrow a = 2$

$b = -2(2) = -4$

$\therefore a - b = 2 - (-4) = 2 + 4 = 6.$

13. A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$, then the largest area of the rectangle is:



(a) 32

(b) 16

(c) $\frac{32}{3}$

(d) 64

(e) 48

$$\text{Area} = A = 2xy$$
$$= 2x(12-x^2)$$
$$\therefore A = 24x - 2x^3$$

$0 \le x \le \sqrt{12} = 2\sqrt{3}$

$$A' = 24 - 6x^2$$

Set $A' = 0 \implies x^2 = \frac{24}{6} = 4$
$\therefore x = 2$ (ignor $x = -2$)

$A'' = -12x \implies A''\big|_{x=2} < 0$
$\therefore$ Max. when $x = 2$
$\therefore$ The largest area $= 24(2) - 2(2)^3$
$= 48 - 16 = 32.$

14. $\lim_{x\to 1}(2-x)^{\tan(\pi x/2)} =$

(a) $e^{2/\pi}$

(b) $e^{-2/\pi}$

(c) $\frac{\pi}{2}$

(d) $e^{\pi/2}$

(e) 2π

Let $y = (2-x)^{\tan\frac{\pi x}{2}}$

$\ln y = \tan\frac{\pi x}{2}\ln(2-x)$

$\ln y = \dfrac{\ln(2-x)}{\cot\frac{\pi x}{2}}$

as $x \to 1$, $\ln y \to \frac{0}{0}$
Using L.R. we have:

$$\lim_{x\to 1}\ln y \overset{L.R}{=} \lim_{x\to 1}\frac{\frac{-1}{2-x}}{-\frac{\pi}{2}\csc^2\frac{\pi}{2}x}$$

$$= \frac{\frac{-1}{1}}{-\frac{\pi}{2}(1)} = \frac{2}{\pi}.$$

$$\therefore \lim_{x\to 1} y = e^{\frac{2}{\pi}}$$

15. If $y = \sin^2\left(\frac{\pi}{9}u\right)$, $u = 2x - 1$. Then $\dfrac{dy}{dx}$ when $x = 2$ is

(a) $\dfrac{\sqrt{3}\,\pi}{9}$

(b) $\dfrac{2\sqrt{3}\,\pi}{9}$

(c) $\dfrac{\sqrt{3}}{9}$

(d) $4\sqrt{3}$

(e) $\dfrac{\sqrt{3}\,\pi}{18}$

$\dfrac{dy}{du} = 2\sin\left(\frac{\pi}{9}u\right).\ \cos\left(\frac{\pi}{9}u\right)\cdot\frac{\pi}{9}$

$\dfrac{du}{dx} = 2$

when $x = 2 \Rightarrow u = 2(2) - 1 = 3$

$\therefore \dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx}$

$\left.\dfrac{dy}{dx}\right|_{x=2} = \frac{2\pi}{9}\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)\cdot(2)$

$= \frac{4\pi}{9}\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \frac{\sqrt{3}\,\pi}{9}.$

16. Let $f(x) = \ln(\cos x)$, $\dfrac{-\pi}{4} \le x \le \dfrac{\pi}{3}$. Which one of the following statements is **TRUE**?

(a) All choices are correct

(b) Increasing on $\left(\dfrac{-\pi}{4}, 0\right)$

(c) Decreasing on $\left(0, \dfrac{\pi}{3}\right)$

(d) $f(x)$ has absolute max. at $x = 0$, and absolute mini. at $x = \dfrac{\pi}{3}$.

(e) $f(x)$ has a local minimum when $x = \dfrac{-\pi}{4}$.

$f'(x) = \dfrac{-\sin x}{\cos x} = -\tan x$

Set $f'(x) = 0 \Rightarrow -\tan x = 0 \Rightarrow x = 0$

$+ + + \ 0 \ - - -$

$-\frac{\pi}{4} \qquad 0 \qquad \frac{\pi}{3}$

$\Rightarrow$ Increasing on $\left(-\frac{\pi}{4}, 0\right)$

Decreasing on $\left(0, \frac{\pi}{3}\right)$

$f(0) = \ln(1) = 0$ is an abs. max.

$f\left(\frac{\pi}{3}\right) = \ln\left(\frac{1}{2}\right) = -\ln 2 \to$ Abs min

$f\left(-\frac{\pi}{4}\right) = \ln\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}\ln 2$

local min. when $x = -\frac{\pi}{4}$

$\Rightarrow$ All choices are Correct

17. The function $y = \dfrac{1}{x^2+3}$ has

(a) One absolute maximum and two inflection points

(b) One absolute maximum and four inflection points

(c) One absolute minimum and two inflection points

(d) One absolute maximum and one inflection points

(e) No maxima or minima and two inflection points.

$\text{Dom} := \mathbb{R} = (-\infty, +\infty)$

$y' = \dfrac{-2x}{(x^2+3)^2}$

Set $y' = 0 \Rightarrow -2x = 0$
$\Rightarrow x = 0$

$+ + + \; 0 \; - - -$ (y'), at 0

$y'' = \dfrac{6(x^2-1)}{(x^2+3)^3}$

Set $y'' = 0 \Rightarrow x = \pm 1$

$\cup \quad \cap \quad \cup$

$+ + + \; 0 \; - - - \; 0 \; + + +$ (y''), at -1 and 1

y has an abs. max. when $x = 0$
" " two inflection points when
$x = -1$ and $x = 1$

18. $\displaystyle\lim_{x\to 0} \frac{\sin^{-1}(2x)}{x} =$

(a) 2

(b) 1

(c) Does not exist

(d) 0

(e) -1

$$\lim_{x\to 0} \frac{\sin^{-1}(2x)}{x} \overset{L.R}{=} \lim_{x\to 0} \frac{\frac{2}{\sqrt{1-4x^2}}}{1}$$

$$= \frac{2}{\sqrt{1}} = 2.$$

19. If $f(x) = \dfrac{h(x)}{g(x)(x^2+1)}$, and $h(1) = -2$, $h'(1) = g(1) = -1$, and $f'(1) = 3$, then $g'(1) =$

(a) $\dfrac{7}{2}$

(b) 6

(c) 7

(d) $\dfrac{3}{2}$

(e) $\dfrac{-1}{6}$

$$f'(x) = \frac{g(x)(x^2+1)h'(x) - h(x)[g(x)(2x) + (x^2+1)g'(x)]}{[g(x)(x^2+1)]^2}$$

$$f'(1) = \frac{g(1)(2)h'(1) - h(1)[g(1)(2) + 2g'(1)]}{(g(1)(2))^2}$$

$$3 = \frac{(-1)(2)(-1) - (-2)[(-1)(2) + 2g'(1)]}{((-1)(2))^2}$$

$$3 = \frac{2 - 4 + 4g'(1)}{4}$$

$$\Rightarrow 4g'(1) = 12 + 2 = 14$$

$$\therefore g'(1) = \frac{14}{4} = \frac{7}{2}.$$

20. If $\displaystyle\lim_{x\to-\infty} \frac{(2 - a^{3/2}x)^2 + x - 1}{(1-x)^n + 3} = -8$, where a and n are integer numbers, then $3n - 2a =$

(a) 10

(b) 2

(c) 3

(d) 1

(e) -2

Since the limit exists $\Rightarrow n = 2$.

$$\text{The limit} = \lim_{x\to-\infty} \frac{a^3x^2 - 4a^{3/2}x + 4 + x - 1}{x^2 - 2x + 1 + 3}$$

$$= \lim_{x\to-\infty} \frac{a^3x^2 - 4a^{3/2}x + x + 3}{x^2 - 2x + 4}$$

$$= \lim_{x\to-\infty} \frac{a^3 - \frac{4a^{3/2}}{x} + \frac{1}{x} + \frac{3}{x^2}}{1 - \frac{2}{x} + \frac{4}{x^2}} = -8$$

$$\Rightarrow a^3 = -8 \Rightarrow a = -2$$

$$\therefore 3n - 2a = 3(2) - 2(-2)$$

$$= 6 + 4$$

$$= 10$$

21. If $y = \sec^{-1} x + \cot^{-1} \sqrt{x^2 - 1}$, $x > 1$. Then $y' =$

(a) 0

(b) $\dfrac{2x}{\sqrt{x^2 - 1}}$

(c) $\dfrac{2}{x^2\sqrt{x^2 - 1}}$

(d) $\dfrac{x}{x^2 - 1}$

(e) $\dfrac{2x}{x^2 - 1}$

$$y' = \frac{1}{x\sqrt{x^2-1}} - \frac{1}{x^2-1+1} \cdot \frac{2x}{2\sqrt{x^2-1}}$$

$$y' = \frac{1}{x\sqrt{x^2-1}} - \frac{x}{x^2\sqrt{x^2-1}}$$

$$= \frac{1}{x\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}} = 0$$

$\therefore\ y' = 0$

22. The function $f(x) = x^{4/3} - x^{1/3}$ has:

(a) Two critical points

(b) No vertical tangents

(c) ~~No~~ A local maximum

(d) ~~No~~ An absolute maximum

(e) One local maximum and one local minimum

$$f'(x) = \frac{4}{3}x^{1/3} - \frac{1}{3}x^{-2/3}$$

$$= \frac{4x^{1/3}}{3} - \frac{1}{3x^{2/3}}$$

$$= \frac{4x-1}{3x^{2/3}}$$

Critical when $f'(x) = 0 \Rightarrow 4x - 1 = 0$
$\Rightarrow x = \frac{1}{4}$

or $f'(x)$ does not exist
$\Rightarrow 4x^{2/3} = 0 \Rightarrow x = 0$

$\therefore$ $f(x)$ has two critical points.

23. Using the linear approximation $(1+x)^k \simeq 1+kx$, the estimated value of $(0.9998)^{50}$ is:

$(0.9998)^{50} = (1+(-0.0002))^{50}$

$\simeq 1 + 50(-.0002)$

$= 1 - 0.01$

$= 0.99$

(a) 0.99

(b) 1.01

(c) 0.995

(d) 0.95

(e) 0.90

24. The number of values in the interval $[0,2]$ that satisfy the hypotheses of the Mean Value Theorem for the function $f(x) = \begin{cases} 3-x^2, & 0 \le x \le 1 \\ \dfrac{2}{x}, & 1 < x \le 2 \end{cases}$ is:

$f(x)$ is Conts. on $[0,2]$

$f'(x) = \begin{cases} -2x, & 0 < x \le 1 \\ \dfrac{-2}{x^2}, & 1 < x < 2 \end{cases}$

$\therefore$ $f(x)$ is diff. on $(0,2)$

$\therefore$ By MVT, there is at least $c \in (0,2)$ such that

$f'(c) = \dfrac{f(2)-f(0)}{2-0} = \dfrac{1-3}{2} = -1$

$\therefore$ Either $-2c = -1 \Rightarrow c = \frac{1}{2} \in (0,1)$ ✓

OR $\dfrac{-2}{c^2} = -1 \Rightarrow c^2 = 2$

$\Rightarrow c = -\sqrt{2} \notin (1,2)$ X

or $c = \sqrt{2} \in (1,2)$ ✓

$\Rightarrow$ There are two values of c

(a) 2

(b) 3

(c) 1

(d) 0

(e) 4

25. The formula for the Riemann sum for the function $f(x) = x + x^2$ by dividing the interval $[0, 1]$ into n equal subintervals and using the right end point of each subinterval is:

(a) $\frac{5}{6} + \frac{1}{n} + \frac{1}{6n^2}$

(b) $1 + \frac{5n}{6} + \frac{1}{6n}$

(c) $\frac{1}{2} + \frac{1}{2n}$

(d) $\frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}$

(e) $\frac{5}{6}$

$$\Delta x = \frac{1-0}{n} = \frac{1}{n}$$

$$c_k = x_k = 0 + \frac{1}{n}k = \frac{k}{n}$$

$$S_n = \sum_{k=1}^{n} f(c_k)\,\Delta x$$

$$= \sum_{k=1}^{n} \left(\frac{k}{n} + \frac{k^2}{n^2}\right)\left(\frac{1}{n}\right)$$

$$= \frac{1}{n}\left\{\frac{1}{n}\sum_{k=1}^{n} k + \frac{1}{n^2}\sum_{k=1}^{n} k^2\right\}$$

$$= \frac{1}{n}\left\{\frac{1}{n}\cdot\frac{n(n+1)}{2} + \frac{1}{n^2}\,\frac{n(n+1)(2n+1)}{6}\right\}$$

$$= \frac{1}{n}\left\{\frac{n}{2} + \frac{1}{2} + \frac{1}{6n}(2n^2+3n+1)\right\}$$

$$= \frac{1}{n}\left\{\frac{n}{2} + \frac{1}{2} + \frac{n}{3} + \frac{1}{2} + \frac{1}{6n}\right\}$$

$$= \frac{1}{n}\left\{\frac{5n}{6} + 1 + \frac{1}{6n}\right\}$$

$$= \frac{5}{6} + \frac{1}{n} + \frac{1}{6n^2}$$

26. If $f(x) = 2x - 3x^{2/3}$, then f is

(a) concave up on $(-\infty, 0)$ and $(0, \infty)$ with no inflection points

(b) concave down $(-\infty, 0)$ and $(0, \infty)$ with no inflection points

(c) concave up on $(-\infty, 0)$ and down on $(0, \infty)$ with 1 inflection point

(d) concave down on $(-\infty, 0)$ and up on $(0, \infty)$ with 1 inflection point

(e) concave up on $(-\infty, 0)$ and down on $(0, 1)$ with 1 inflection points.

$$f'(x) = 2 - 2x^{-1/3}$$

$$f''(x) = \frac{2}{3}x^{-4/3} = \frac{2}{3\sqrt[3]{x^4}}$$

$f''(x) \neq 0$

$f''(x)$ does not exist when $x = 0$

f'': $+ + + \; 0 \; + + +$ (at 0)

$\therefore$ $f(x)$ is concave up on $(-\infty, 0)$ and $(0, \infty)$ with no inflection points

27. Let $S(t) = \dfrac{t}{v(t)}$ be the position function of a particle moving along a coordinate line, and $v(t)$ is the velocity of the particle at time t. If $v(2) = 3\,m/s$, then the particle acceleration, when $t = 2$ (in m^2/s), is:

(a) -12

(b) Cannot be evaluated

(c) 15

(d) $\dfrac{3}{2}$

(e) -3

$S'(t) = \dfrac{v(t)\cdot 1 - t\cdot v'(t)}{(v(t))^2}$

but $S'(t) = v(t)$, $v'(t) = a(t)$

$\therefore\ v(t) = \dfrac{v(t) - t\,a(t)}{(v(t))^2}$

when $t=2$

$\dfrac{3}{1} = \dfrac{3 - 2a(2)}{(3)^2}$

$3 - 2a(2) = 27$

$2a(2) = -24$

$a(2) = -12\ m^2/s$.

28. A can that must ~~hod~~ hold $128ml$ is made in the shape of right circular cylinder. The dimensions of the can that minimizes the surface area is:(Volume $= \pi$ (radius)$^2 \cdot$ hight).

(a) radius $= \dfrac{4}{\sqrt[3]{\pi}}$ and hight $\dfrac{8}{\sqrt[3]{\pi}}$

(b) radius $= \dfrac{4}{\sqrt[3]{\pi}}$ and hight $\dfrac{4}{\sqrt[3]{\pi}}$

(c) radius $= \dfrac{8}{\sqrt[3]{\pi}}$ and hight $\dfrac{8}{\sqrt[3]{\pi}}$

(d) radius $= \dfrac{8}{\sqrt[3]{\pi}}$ and hight $\dfrac{4}{\sqrt[3]{\pi}}$

(e) radius $= \dfrac{8}{\sqrt[3]{\pi}}$ and hight $\dfrac{16}{\sqrt[3]{\pi}}$

Let S be the surface area

r: radius, h: hight.

$S = 2\pi r^2 + 2\pi r h$

Since $\pi r^2 h = 128 \Rightarrow h = \dfrac{128}{\pi r^2}$

$\therefore\ S = 2\pi r^2 + 2\pi r\cdot \dfrac{128}{\pi r^2}$

$= 2\pi r^2 + \dfrac{256}{r}$.

$S' = 4\pi r - \dfrac{256}{r^2} \overset{\text{set}}{=} 0$

$4\pi r^3 = 256 \rightarrow r^3 = \dfrac{64}{\pi}$

$\therefore\ r = \dfrac{4}{\sqrt[3]{\pi}}$

$S'' = 4\pi + \dfrac{512}{r^3}\Big|_{r=\frac{4}{\sqrt[3]{\pi}}} > 0$

$\therefore$ minimum when $r = \dfrac{4}{\sqrt[3]{\pi}}$

$\therefore\ h = \dfrac{128}{\pi\cdot 16}\cdot \pi^{2/3} = \dfrac{8}{\pi^{1/3}} = \dfrac{8}{\sqrt[3]{\pi}}$.