King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 201 Major Exam I
The Third Semester of 2014-2015 (143)
Time Allowed: 120 Minutes

Name: ___________________________     ID#: ________________________
Section/Instructor: _____________________     Serial #: ________________________

- Mobiles and calculators are not allowed in this exam.
- Provide all necessary steps required in the solution.

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Q:1 Consider the parametric equations $x = 2 + \sin t$, $y = \cos t + 1$.

(a) (5 points) Eliminate the parameter to find a cartesian equation.

(b) (5 points) Sketch the curve for $0 \leq t \leq \pi$ and mark the direction in which the curve is traced as $t$ increases.
Q:2  (a) (7 points) Find the equation of tangent line to the curve \( t = \ln(x - t), \quad y = t e^t \) at \( t = 0 \).

(b) (7 points) Find the length of the curve

\[
\begin{align*}
x &= 2t - 2\sin t, & y &= 2 - 2\cos t, & 0 \leq t \leq 2\pi.
\end{align*}
\]
Q:3 (a) (6 points) Write the polar equation $r = (\ln r - \ln \cos \theta) \csc \theta$ in cartesian coordinates.

(b) (6 points) Graph the sets of points whose polar coordinates satisfy the following conditions

$$1 \leq r \leq 2 \text{ and } \frac{2\pi}{3} \leq \theta \leq \frac{5\pi}{6}.$$
Q:4 (a) (6 points) Identify the symmetries of the curve $r^2 = 4 \cos \theta$.

(b) (6 points) Find the slope of the curve $r = 1 + \sin \theta$ at $\theta = \frac{\pi}{3}$. 
Q:5 (14 points) Find the area of the region that lies inside both curves $r = \cos 2\theta$ and $r = \sqrt{3} \sin 2\theta$ for $0 \leq \theta \leq \frac{\pi}{2}$.
Q:6 (a) (6 points) Find an equation of the sphere that passes through the point $(2, -4, 3)$ and has center $(1, 2, 5)$. Describe the intersection of this sphere with the $xz$-plane.

(b) (6 points) If the angle between two unit vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, then find the value of $|3\vec{a} - 2\vec{b}|$. 

Q:7 (6 points) Find the vector projection of \( \vec{a} = \langle 1, 1, 1 \rangle \) onto \( \vec{b} = \langle 2, 3, 4 \rangle \) and the scalar component of \( \vec{a} \) in the direction of \( \vec{b} \).

(b) (8 points) Find a unit vector perpendicular to the plane P(1, -1, 0), Q(2, 1, -1) and R(-1, 1, 2).
Q:8 (10 points) Find the volume of the parallelepiped determined by the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{AD}$ where $A(1, 0, 0)$, $B(0, 2, 0)$, $C(0, 0, 3)$, $D(0, 1, 3)$. 