

Question 1 Solve

a) $(\cos(3x) + \ln y)dx + \left(\frac{x}{y} - e^{2y}\right)dy = 0$

b) $y' = 2xy + 3x^2e^{x^2}$ with $y(0) = 5$

Question 2 Find the general solution of:

a) $(D^2 + 2D + 5)^2 y = 0$

b) $y''' + 3y'' - 9y' - 27y = 0$

Question 3 Solve the system of first order linear DE: $X' = AX$ where

$$A = \begin{bmatrix} 5 & 5 & 2 \\ -6 & -6 & -5 \\ 6 & 4 & 3 \end{bmatrix}.$$

Question 4 Given that $y_c = C_1 \cos(3x) + C_2 \sin(3x)$ is a general solution of $y'' + 9y = 0$. Find the general solution of: $y'' + 9y = \sec(3x)$.

Question 5 Consider the following nonhomogeneous system of DE:

$$X' = \begin{pmatrix} 5 & -1 & 1 \\ 1 & 3 & 0 \\ -3 & 2 & 1 \end{pmatrix} X + \begin{pmatrix} 1 - 5t \\ -t \\ 3t \end{pmatrix} \quad (1)$$

where $X_1 = e^{3t} \begin{pmatrix} 2t + 1 \\ t^2 + t \\ t^2 - 3t \end{pmatrix}$ is a solution of associated homogeneous system of DE. Verify that

$$X = C_1 X_1 + e^{3t} \left[C_2 \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} + C_3 \begin{pmatrix} 2 \\ 2t + 1 \\ 2t - 3 \end{pmatrix} \right] + \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix}$$

form a general solution of (1).

Question 6 Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ -1 & 2 & -3 & 0 \\ -5 & -2 & 3 & 2 \\ 1 & 3 & 2 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 5 & 0 & 3 & 1 \\ 3 & 2 & 2 & -3 \\ 2 & -2 & 5 & 4 \\ 7 & 0 & 2 & 1 \end{bmatrix}$$

where $|A| = 8$ and $|B| = -16$.

a) Show that the following system has a unique solution:

$$A^{-1}B \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad (2)$$

b) Solve the system (2) for z only.

Question 7

a) Solve $X' = AX$ where $A = \begin{bmatrix} 1 & 3 & 7 & 0 \\ 0 & -6 & 5 & 0 \\ 0 & -5 & 4 & 0 \\ 0 & -6 & -14 & 1 \end{bmatrix}$.

b) Check if A is diagonalizable or not. If so, find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

Question 8 Let $W = \left\{ \begin{pmatrix} x \\ e^x - 1 \end{pmatrix} \in \mathbb{R}^2 \right\}$. Show that W is not a subspace of \mathbb{R}^2 .