Sunday, June 21, 2015  
Allowed Time: 1 Hours

Instructions:

1. Write neatly and legibly. You may lose points for messy work.

2. Show all your work. No points for answers without justification.

3. Calculators and Mobiles are not allowed.

4. Make sure that you have 3 different problems (3 pages + cover page).

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</table>
1) Use Green’s Theorem to find the work done by the force
\[ \mathbf{F} = -xy^2 \mathbf{i} + x^2 y \mathbf{j} \]
around the given closed curve.
2) Evaluate the line integral

\[ \int_C 8y \, dx + 7z \, dy + 7x \, dz \]

on the given curve between (0,0,0) to (6,8,5).
3) Consider the vector field \( \mathbf{F} = y \mathbf{i} + x \mathbf{j} \).

i) Show that \( \mathbf{F} \) is conservative.

ii) Find a potential \( \phi(x, y) \) whose gradient is the vector field \( \mathbf{F} \).

iii) Evaluate the line integral

\[
\int_C y \, dx + x \, dy
\]

along the curve \( C \) between \((-3, -3)\) and \((4,4)\).
QUESTION 1

1. Let \( F = < 4zy, 5xz, 4xy > \)
   Find \( \text{div} (\text{curl} \ F) \)

10 points

QUESTION 2

1. Let \( F = < 6zy, 6xz, 4xy > \)
   Find \( || \text{curl} \ F ||_{(1,2,3)} \) =

10 points

QUESTION 3

1. Let \( f = \frac{3zxy}{x} + \frac{8xz}{y} + \frac{8xy}{z} \)
   Find \( \text{curl} (\text{grad} \ f) \) =

10 points
**QUESTION 4**

1. Let \( f = \frac{4yz}{x} + \frac{3xz}{y} + \frac{4xy}{z} \)

   Find
   \[
   \nabla \cdot \left( \nabla f \right) \big|_{(1,1,1)} =
   \]

   10 points

**QUESTION 5**

1. Find the directional derivative of
   \( f(x,y) = (5xy + 4)^2 \)
   at the point (2,4) in the direction of (4,3)

   10 points

**QUESTION 6**

1. Let \( g(x,y,z) = xyz \) then the minimum value of
   \( D_u g(0,1,-1) \) is 1

   - True
   - False

   10 points

**QUESTION 7**

1. The velocity vector field \( F(x,y,z) = y \mathbf{i} + x \mathbf{j} + z \mathbf{k} \)
   for a fluid is irrotational

   - True
   - False

   10 points
QUESTION 8

1. The vector field \( F(x,y) = x \, i + y \, j \) is conservative

   [ ] True
   [ ] False

   10 points

QUESTION 9

1. The function \( \phi(x,y) = x^2 - y^2 \) is a potential function for \( F(x,y) = 2x \, i + y \, j \)

   [ ] True
   [ ] False

   10 points

QUESTION 10

1. Let \( r(t) \) be a vector function that satisfies
   \( r''(t) = <0, -5\sin t, -\cos t> \)
   and
   \( r(0) = (0,0,1) \)
   and
   \( r'(0) = (3,5,0) \)
   then
   \[ || r(0) || = \]

   [ ]

   10 points
**QUESTION 11**

1. The parametric curve of 
\[(x-1)^2/3^2 + (y+2)^2/8^2 = 1\]
is 
\[x(t) = a \cos t + c \]
\[y(t) = b \sin t + d \]
\[0 < t < \pi \]
then 
\[a + b + c + d = \]

10 points

**QUESTION 12**

1. The arclength of the curve traced by the vector function 
\[r(t) = (3\cos t) \mathbf{i} + (3\sin t) \mathbf{j} + (7t) \mathbf{k}, 0 < t < 2\pi \]
is equal to 
\[(\text{note: use 4 decimals and } \pi = 3.14159)\]

10 points

*Click Save and Submit to save and submit. Click Save All Answers to save all answers.*