King Fahd University of Petroleum & Minerals  
Department of Mathematics and Statistics  
MATH 302  
Final Exam  
2014-2015 (143)

Thursday, August 13, 2015  
Allowed Time: 3 Hours

Name: _______________________________________________________

ID Number: ____________________  
Serial Number: ________________

Section Number: ____________  
Instructor’s Name: Ahmad Al-Dweik

Instructions:

1. Write neatly and legibly. You may lose points for messy work.

2. Show all your work. No points for answers without justification.

3. Programmable Calculators and Mobiles are not allowed.

4. Make sure that you have 7 different problems (7 pages + cover page).

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Q1. Let \( A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \).

(a) Find a matrix \( P \) that diagonalizes \( A \) and the diagonal matrix \( D \) such that \( D = P^{-1}AP \)

(b) Find \( A^{10} \)  

(Hint: Use the fact that if \( m \) is a positive integer, then \( A^m = PD^mP^{-1} \))
Q2. Let D be the region bounded by the paraboloid \( z = x^2 + y^2 \) and the plane \( z = 1 \).

Use the **divergence theorem** to find the outward flux \( \iiint_S (\mathbf{F} \cdot \mathbf{n}) \, dS \) of the vector field \( \mathbf{F} = y \mathbf{i} + x \mathbf{j} + z^2 \mathbf{k} \), where \( S \) is the boundary of D.
Q3. Let $f(z) = Re(z)|z|^2$. We denote by $x = Re(z)$, $y = Im(z)$, $u(x, y) = Re(f(z))$ and $v(x, y) = Im(f(z))$.

(a) Find all $(x, y)$ at which $u, v$ satisfy Cauchy-Riemann Equations.

(b) Show that $f(z)$ is differentiable at $z_0 = 0$. 
Q4. Find all complex numbers $z$ such that $\sin z = 2$. 
Q5. Expand $f(z) = \frac{2}{z^2 - 4z + 3}$ as Laurent series valid for $0 < |z - 1| < 2$. 
Q6. Use the generalized Cauchy integral formula (for higher derivatives) to evaluate the integral \( \oint_C \frac{z^3}{(z-i)(z+i)^3} \, dz \) where \( C \) is the positively oriented circle \(|z + i| = 1\).
Q7. Use the residue theorem to evaluate:

(a) \[ \oint_C \frac{e^z}{z(z-2)^3} \, dz \]

where \( C \) is the positively oriented circle \(|z| = 3\).

(b) \[ \oint_C \exp \left( \frac{5}{z} \right) \, dz \]

where \( C \) is the positively oriented circle \(|z - 1| = 3\).