Q.No.1: A manufacturer of automobiles conducted a market survey. Eighty percent of the customers want better fuel efficiency, while 55% want a vehicle navigation system and 45% want both features.

a. Find the probability that a person wants either better fuel efficiency or a vehicle navigation system.

b. Find the probability that a person wants better fuel efficiency but not a vehicle navigation system.

c. Find the probability that a person wants a vehicle navigation system given that he also wants a better fuel efficiency.

d. Let the event $A$: the customers want better fuel efficiency, $B$: the customers want a vehicle navigation system, are the two events independent? Explain using probability.
Q.No.2: In a semiconductor manufacturing process, three wafers from a lot are tested. Each wafer is classified as pass or fail. Assume that the probability that a wafer passes the test is 0.8 and that wafers are independent. Determine the probability mass function of the number of wafers from a lot that pass the test. Also find the probability of getting one wafer that passes the test.

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\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A \cap B') + P(A \cap B) + P(A' \cap B) \]

\[ P(A \cup B) = 1 - P(A \cap B)' = 1 - P(A' \cap B') \quad \text{and} \quad P(A \cap B)' = P(A' \cap B') \]

\[ P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0 \quad \text{and} \quad P(A \cap B) = P(A). P(B | A) = P(A | B). P(B) \]

\[ P(B_i | A) = \frac{P(A | B_i). P(B_i)}{\sum_{i=1}^{k} P(A | B_i). P(B_i)}, \quad P(A) \neq 0 \]

\[ \mu = E(X) = \sum x f(x); \quad E(X^2) = \sum x^2 f(x) \quad \text{and} \quad \sigma^2 = E(X - \mu)^2 = E(X^2) - \mu^2 \]

Discrete Uniform Distribution: \( f(x) = \frac{1}{n}; \quad x = x_1, x_2, \ldots, x_n; \quad \mu = \frac{x_n+x_1}{2}; \quad \sigma^2 = \frac{(x_n-x_1+1)^2-1}{12} \)

Binomial Distribution: \( f(x) = \binom{n}{x} p^x(1-p)^{n-x}; \quad x = 0,1,\ldots, n; \quad \mu = np; \quad \sigma^2 = np(1-p) \)

Geometric Distribution: \( f(x) = p(1-p)^{x-1}; \quad x = 1,2,\ldots; \quad \mu = 1/p; \quad \sigma^2 = (1-p)/p^2 \)

Hypergeometric Distribution: \( f(x) = \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}; \quad x = \max\{0,n+K-N\} \text{ to } \min\{K,n\}; \mu = np; \)

\[ \sigma^2 = np(1-p) \frac{N-n}{N} \frac{N-1}{N-1}; \quad p = \frac{K}{N} \]

Poisson Process: \( f(x) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}; \quad x = 0,1,\ldots; \mu = \lambda t; \quad \sigma^2 = \lambda t \)

\[ F(b) = P(X \leq b) = \int_{-\infty}^{b} f(x) \, dx \quad \text{and} \quad P(a < X < b) = \int_{a}^{b} f(x) \, dx = F(b) - F(a) \]

\[ \mu = E(X) = \int_{-\infty}^{\infty} x f(x) \, dx; \quad E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) \, dx \quad \text{and} \quad \sigma^2 = E(X - \mu)^2 = E(X^2) - \mu^2 \]

Continuous Uniform Distribution: \( f(x) = \frac{1}{x_n-x_1}; \quad x_1 \leq x \leq x_n; \quad \mu = \frac{x_n+x_1}{2}; \quad \sigma^2 = \frac{(x_n-x_1)^2}{12} \)

Exponential Distribution: \( f(x) = \lambda e^{-\lambda x}; \quad x > 0; \quad \mu = \frac{1}{\lambda}; \quad \sigma^2 = \frac{1}{\lambda^2} \)