A rocket motor is manufactured by bonding together two types of propellants, an igniter and a sustainer. The shear strength of the bond $Y$ (measured in psi) is thought to be a linear function of the age of the propellant $X$ (measured in weeks) when the motor is cast. Twenty observations gave the following summary quantities.

\[ n = 20, \quad \sum y_i = 42648.15, \quad \sum x_i = 266.75, \quad \sum y_i^2 = 92642656, \quad \sum x_i^2 = 4672.44, \quad \sum x_i y_i = 527619.9 \]

a) Calculate $S_{XX}, S_{YY}$ and $S_{XY}$

b) Estimate the degree of linear correlation between and the shear strength of the bond and the age of the propellant. Interpret this quantity
c) Find the estimated regression line. What are your assumptions?

Assumptions:


d) What change in mean shear strength of the bond would be expected for a 1 week change in the age of the propellant?

e) Estimate the error variance.
f) At 5% level of significance, test that the higher the age of the propellant, the larger the shear strength of the bond.

g) Calculate the coefficient of determination and interpret it.

Interpretation: ____________________________________________________________
h) Estimate the mean shear strength of the bond when the age of the propellant is 20 weeks, using 95% confidence level.

\[ S_{XX} = \sum x^2 - \frac{1}{n} (\sum x)^2, \quad S_{YY} = \sum y^2 - \frac{1}{n} (\sum y)^2, \quad S_{XY} = \sum xy - \frac{1}{n} (\sum y)(\sum x) \]

\[ Y = \beta_0 + \beta_1 X + \epsilon, \quad \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X, \quad e = |Y - \hat{Y}|, \quad \hat{\beta}_1 = \frac{s_{xy}}{s_{xx}}, \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \]

\[ SST = SSR + SSE, \quad SSR = \hat{\beta}_1 s_{xy}, \quad SST = s_{yy}, \quad SSE = SST - SSR, \quad \hat{\sigma}^2 = \frac{SSE}{n-2} \]

\[ r = \frac{s_{xy}}{\sqrt{s_{xx}s_{yy}}} = \hat{\beta}_1 \sqrt{\frac{s_{xx}}{s_{yy}}} \quad \text{and} \quad R^2 = \frac{\text{Explained Variation}}{\text{Total Variation}} = \frac{SSR}{SST} \]

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Formula</th>
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<tbody>
<tr>
<td>( \hat{\beta}<em>0 \pm \frac{t_a}{\sqrt{n-2}} \sqrt{\hat{\sigma}^2 \left[ \frac{1}{n} + \frac{x^2}{s</em>{xx}} \right]} )</td>
<td>( T = \frac{\hat{\beta}<em>0 - \beta_0}{\sqrt{\hat{\sigma}^2 \left[ \frac{1}{n} + \frac{x^2}{s</em>{xx}} \right]} } )</td>
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<tr>
<td>( \hat{\beta}<em>1 \pm \frac{t_a}{\sqrt{n-2}} \sqrt{\hat{\sigma}^2 \frac{1}{s</em>{xx}}} )</td>
<td>( T = \frac{\hat{\beta}<em>1 - \beta_1}{\sqrt{\hat{\sigma}^2 \frac{1}{s</em>{xx}}} } )</td>
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<td>( \hat{\gamma}<em>0 \pm \frac{t_a}{\sqrt{n-2}} \sqrt{\hat{\sigma}^2 \left[ 1 + \frac{1}{n} + \frac{(x_0-x)^2}{s</em>{xx}} \right]} )</td>
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