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AS 201: Financial Mathematics - Exam II

Sunday, November 22nd, 2015 (5 pm – 7 pm)

Exam guidelines:

- 1 – The exam is composed of 20 Questions (5 points each).
- 2 – Please show your work completely
- 3 – This is an individual work. If you are seen cheating, then you will be asked to leave the examination room and will be given ZERO.
- 4 – Only the following items are allowed inside the examination room:
 - SOA approved calculator (all calculators must be reset)
 - Pen or pencil
 - Sharpener
 - Eraser
- 5 – No breaks are allowed once the exam has started. No one is allowed to leave the examination room to go to the bathroom.
- 6 – Exam will end after three hours exactly. No extra time will be given.
- 7 – The exam proctor will not answer any questions and students are not allowed to ask any questions, neither.
- 8 – Cell phones must be turned off and given to the exam proctor



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Exercise # 1

The following is a yield curve table with given maturities and spot rates.

Year	Spot Rate (%)
1	6
2	6.5
3	X
4	Y
5	8

The 3-year forward rate is 9.5% and the 4-year forward rate is 10%. Calculate the annual effective rate of return for Zero coupon bonds with maturity of 3 years.

- A) 6.6%
- B) 6.8%
- C) 7.4%
- D) 7.5%
- E) 7.9%



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Exercise # 2

Using the following table to represent spot rates, the price of a 5-year 1000 bond with 7% annual coupon is calculated. What would be an equivalent constant yield-to-maturity of the bond with the calculated price?

Term (years)	Yield (%)
1	4.75
2	5
3	5.25
4	5.5
5	6

- A) 5.1 %
- B) 5.3 %
- C) 5.5 %
- D) 5.7 %
- E) 5.9 %



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Exercise # 3

An investment pays 2000 in 3 years and 4000 at the end of the 4th year. If it was purchased to yield an annual rate of 7.5%, what would be the modified duration?

- A) 3.35
- B) 3.4
- C) 3.5
- D) 3.65
- E) 3.7



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Exercise # 4

An association has an initial balance of 200 on January 1 and also had quarterly deposits of 25 on the last day of every 3 months after January 1. The association had a withdrawal of 30 on February 28th and 60 on June 30th, and ended with a balance of 250 on December 31st. Calculate their dollar weighted rate of return

- A) 18%
- B) 19%
- C) 20%
- D) 21%
- E) 22%



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Exercise # 5

If the investment year rates are described by the function: $1 + i_t^y = (1.2 - 0.01t)^{1+0.02y}$,
for $1 \leq t \leq 5$ and $1 \leq y \leq 10$. What would be the accumulated value for an investment
of 2400 made in year $y = 2$, left in the fund, and sold 4 years later

- A) 4693
- B) 4711
- C) 4732
- D) 4795
- E) 4809



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Exercise # 6

A proposed project has been presented that requires an initial investment of 5 million and a second investment of 3 million at the beginning of year 3. It will generate a return of 2 million at the end of 3 years and returns 4 million at the end of years 5 and 6.

What is the Internal rate of return for the project?

- A) 5.2%
- B) 5.3%
- C) 5.4%
- D) 5.5%
- E) 5.6%



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Exercise # 7

Suppose a yield curve for spot rates is given by the following equation:

$S_t = 0.08 - 0.001t + 0.002t^2$. What would be the effective annual forward interest rate for a loan originating at time $t = 4$, with a term of 3 years.

- A) 7%
- B) 11%
- C) 17%
- D) 21%
- E) 26%



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Exercise # 8

What is the Macaulay duration of a 5 year 2000 par value bond with 8% annual coupon and an effective annual rate of interest equal to 7%

- A) 4
- B) 4.1
- C) 4.2
- D) 4.3
- E) 4.5



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Exercise # 9

Consider the following information about an annual coupon bond:

Current price = 1000

The derivative of the current price of the bond with respect to the yield to maturity is -700

The yield to maturity, 8%, is an annual effective rate

Calculate the Duration of the bond.

- A) 7
- B) 7.49
- C) 7.56
- D) 7.69
- E) 8



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Exercise # 10

Consider a common stock that pays constant dividends at the end of each year into perpetuity. Effective rate of interest is 10%.

Calculate the duration of the common stock.

- A) 7
- B) 9
- C) 11
- D) 19
- E) 27



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Exercise # 11

Consider a common stock that pays dividends at the end of each year into perpetuity.

Dividends increase by 2% each year. Effective rate of interest is 5%.

Calculate the duration of the common stock.

- A) 27
- B) 35
- C) 44
- D) 52
- E) 58



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Exercise # 12

A 3-year bond has a face value of 1000 is sold at par. The bond pays a coupon of 100 at the end of each of the next 3 years. Find the bond's Macaulay duration when valued using an annual effective rate of 20%.

- A) 2.61
- B) 2.7
- C) 2.77
- D) 2.89
- E) 3



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Exercise # 13

Consider the following portfolio of bonds:

The 1st bond has semi-annual coupons at 4%, a duration of 21.46 years and bought at 980

The 2nd bond is a 15-year bond with duration 12.35 years and bought at 1015

The 3rd bond has a duration of 16.67 years and bought at 1000

Calculate the duration of the portfolio at the time they were bought.

- A) 16.62
- B) 16.67
- C) 16.72
- D) 16.77
- E) 16.82



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Exercise # 14

You enter into an engagement with a colleague to pay him 10000 at the end of each year for 2 years. In order to exactly match this obligation, you buy a combination of the following 2 bonds:

- 1-year 4% annual coupon bond with yield 5%
- 2-year 6% annual coupon bond with yield 5%

Find the total cost of your purchase.



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Exercise # 15

Consider an 8-year 100 par value bond with 10% annual coupons and effective rate of interest equal to 8%. Calculate the Macaulay duration.



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Exercise # 16

A company must pay liabilities of 20000 and 40000 at the end of years 1 and 2, respectively. The only investments available to the company are the following two annual coupon bonds, which are both redeemable at par

Bond	Maturity (Years)	Annual coupon (%)	Effective annual yielded (%)	Par value
Bond A	1	5	4	X
Bond B	2	6	4.5	1000

In order to match the liabilities exactly, they must buy an amount of 16.1 of Bond A. What is the total cost of purchasing the bonds to match the liabilities?



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Exercise # 17

Which of the following statements about immunization strategies are true?

- I. To achieve immunization, the convexity of the assets must equal the convexity of the liabilities
 - II. The full immunization technique is designed to work for any change in the interest rate
 - III. The theory of immunization was developed to protect against adverse effects created by changes in interest rates
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- A) none
 - B) I and II only
 - C) I and III only
 - D) II and III only
 - E) Neither of the above answers are correct



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Exercise # 18

The dividends of a common stock are expected to be 1 at the end of each of the next 5 years and 2 for each of the following 5 years. The dividends are expected to grow at a fixed rate of 2% per year thereafter. Assume an annual effective interest rate of 6%. Calculate the price of this stock.



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Exercise # 19

A stock is expected to have a dividend of 5 in one year. Each annual subsequent dividend is expected to be 2% larger than the preceding one. At a valuation interest rate of 4%, calculate the modified duration of the stock.



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Exercise # 20

Assume that you must pay liabilities of 1000 due 6 months from now and another 1000 due one year from now.

The only 2 available investments you have access to are:

- I. a 6-month bond with face amount of 1000, a 8% nominal annual coupon rate convertible semiannually and a 6% nominal annual yield rate convertible semiannually.
- II. A one-year bond with face amount of 1000, a 5% nominal annual coupon rate convertible semiannually and a 7% nominal annual yield convertible semiannually.

Calculate the annual effective yield rate for investment in the bonds required to exactly match the liabilities.