Q1. [10 points]

1. (3 points) Evaluate \( \lim_{n \to \infty} \frac{5^n}{n!} \).

2. (3 points) Evaluate \( \int_0^1 \int_x^1 \frac{1}{1+y^2} dy dx \).

3. (4 points) Let \( f(x) = x^4 e^x \). Determine the \( n^{th} \) derivative of \( f \) at \( x = 0 \).
Q2]...[10 points] An actuary is studying the prevalence of three health risk factors, denoted by A, B, and C, within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B, is $\frac{1}{3}$. What is the probability that a woman has none of the three risk factors, given that she does not have risk factor A?
Q3]...[10 points] A box contains 4 red balls and 6 white balls. A sample of size 3 is drawn without replacement from the box. What is the probability of obtaining 1 red and 2 white balls, given that at least 2 of the balls in the sample are white?
Q4]...[10 points] Let $X_1, X_2,$ and $X_3$ be three independent, identically distributed random variables each with density function

$$f(x) = \begin{cases} 
3x^2 & \text{if } 0 \leq x \leq 1 \\
0 & \text{otherwise}
\end{cases}$$

Let $Y = \max\{X_1, X_2, X_3\}$. Find $P(Y > \frac{1}{2})$. 

Q5]...[10 points] Let the distribution function of $X$ for $x > 0$ be

$$F(x) = 1 - \sum_{k=0}^{3} \frac{x^k e^{-x}}{k!}.$$

What is the density function of $X$ for $x > 0$?
Q6][10 points] Let $X_1, X_2, X_3$ be a random sample from a discrete distribution with probability function

$$p(x) = \begin{cases} 
\frac{1}{3} & \text{if } x = 0 \\
\frac{2}{3} & \text{if } x = 1 \\
0 & \text{otherwise}
\end{cases},$$

Determine the moment generating function, $M(t)$, of $Y = X_1X_2X_3$. 

GOOD LUCK