King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 102 - Exam II - Term 151

Duration: 90 minutes

the KEY

Name:____________________ ID Number:____________________

Section Number:____________________ Serial Number:____________________

Class Time: ____________________ Instructor's Name:____________________

Instructions:

1. Calculators and Mobiles are not allowed.
2. Write legibly.
3. Show all your work. No points for answers without justification.
4. Make sure that you have 7 pages of problems (Total of 7 Problems)

Question Number	Points	Maximum Points
1		16
2		22
3		10
4		10
5		10
6		12
7		20
Total		100

1. Evaluate the following integrals:

(a) **[8 points]** $\int \frac{1}{\sqrt{1+x^2}}\,dx$

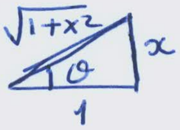
1 Let $x = \tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Then

1 $dx = \sec^2\theta \, d\theta$

1 $\sqrt{1+x^2} = \sqrt{1+\tan^2\theta} = \sqrt{\sec^2\theta} = |\sec\theta| = \sec\theta$ as $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Now $\int \frac{1}{\sqrt{1+x^2}}\,dx = \int \frac{1}{\sec\theta} \sec^2\theta \, d\theta$

$= \int \sec\theta \, d\theta$ 1



$= \ln|\sec\theta + \tan\theta| + C$ 2

$= \ln|\sqrt{1+x^2} + x| + C$ 2

(b) **[8 points]** $\int 3^{x^2+\log_3 x}\,dx$

$= \int 3^{x^2} \cdot 3^{\log_3 x}\,dx$ 2

$= \int 3^{x^2} \cdot x \, dx$ 2

$u = x^2 \Rightarrow du = 2x\,dx$

$= \frac{1}{2}\int 3^u \, du$ 1

$= \frac{1}{2} \frac{3^u}{\ln 3} + C$ 2

$= \frac{1}{2\ln 3} 3^{x^2} + C$ 1

2. (a)[**8 points**] Find a real number a such that $\coth a = \frac{5}{3}$.

$$\coth a = \frac{5}{3} \Longrightarrow \frac{\cosh a}{\sinh a} = \frac{5}{3} \quad 1$$

$$\Longrightarrow \frac{e^{a} + e^{-a}}{e^{a} - e^{-a}} = \frac{5}{3} \quad 2$$

$$\Longrightarrow \frac{e^{2a}+1}{e^{2a}-1} = \frac{5}{3} \quad 2 \quad \Longrightarrow 3e^{2a} + 3 = 5e^{2a} - 5$$

$$\Longrightarrow e^{2a} = 4 \quad 1$$

$$\Longrightarrow 2a = \ln 4 \quad 1$$

$$\Longrightarrow a = \frac{1}{2}\ln 4 \quad 1$$

$$\text{or} \Longrightarrow a = \ln\sqrt{4} = \ln 2$$

(b) [**8 points**] Let $f(x) = \tanh^2 x$. Find $f'(\ln 2)$. (Write your answer as a rational number).

2 . $f'(x) = 2\tanh x \cdot \operatorname{sech}^2 x$

. $f'(\ln 2) = 2 \cdot \frac{3/4}{5/4} \cdot \frac{1}{\left(\frac{5}{4}\right)^2}$

2 $= \frac{96}{125}$

. $\sinh x = \frac{e^{x} - e^{-x}}{2} \Longrightarrow \sinh(\ln 2) = \frac{2 - \frac{1}{2}}{2} = \frac{3}{4}$ 2

. $\cosh x = \frac{e^{x} + e^{-x}}{2} \Longrightarrow \cosh(\ln 2) = \frac{2 + \frac{1}{2}}{2} = \frac{5}{4}$ 2

(c)[**6 points**] Evaluate $\int \frac{\sinh x}{1 + \cosh x}\, dx$

Let $u = 1 + \cosh x$. Then $du = \sinh x \, dx$. 1 + 1

$$\int \frac{\sinh x}{1+\cosh x}\,dx = \int \frac{1}{u}\,du \quad 1$$

$$= \ln|u| + C \quad 2$$

$$= \ln|1+\cosh x| + C \quad 1$$

$$= \ln(1+\cosh x) + C, \text{ as } 1+\cosh x > 0 \text{ for all } x$$

3. [**10 points**] Evaluate $\int 4 \sin^4 t \, dt$.

$$= \int 4 \cdot (\sin^2 t)^2 \, dt \qquad 1$$

$$= \int 4 \cdot \left(\frac{1-\cos(2t)}{2}\right)^2 dt \qquad 2$$

$$= \int 1 - 2\cos(2t) + \cos^2(2t) \quad dt \qquad 2$$

$$= \int 1 - 2\cos(2t) + \frac{1+\cos(4t)}{2} \quad dt \qquad 2$$

$$= \int \frac{3}{2} - 2\cos(2t) + \frac{1}{2}\cos(4t) \quad dt$$

$$= \frac{3}{2} t - \sin(2t) + \frac{1}{8}\sin(4t) + C \qquad 1+1+1$$