

King Fahd University of Petroleum and Minerals  
Department of Mathematics and Statistics

**Math 102**  
**Final Exam**  
**Term 151**  
**Monday 28/12/2015**  
**Net Time Allowed: 180 minutes**

**MASTER VERSION**

1.  $\int_1^4 \frac{1}{\sqrt{w}(1+\sqrt{w})^2} dw =$

$$u = 1 + \sqrt{w} \Rightarrow du = \frac{1}{2\sqrt{w}} dw$$

$$, w=1 \Rightarrow u = 1 + \sqrt{1} = 2$$

$$, w=4 \Rightarrow u = 1 + \sqrt{4} = 3$$

(a)  $\frac{1}{3}$

(b)  $-\frac{2}{3}$

(c)  $\frac{1}{9}$

(d)  $\frac{3}{5}$

(e)  $\frac{1}{4}$

$$= 2 \int_2^3 \frac{1}{u^2} du = 2 \cdot \left. -\frac{1}{u} \right|_{u=2}^3$$

$$= 2 \cdot \left( -\frac{1}{3} + \frac{1}{2} \right)$$

$$= 2 \cdot \frac{-2+3}{6}$$

$$= 2 \cdot \frac{1}{6} = \frac{1}{3}$$

2.  $\int (3 - \tan x)^2 dx = \int 9 - 6 \tan x + \tan^2 x dx$

(a)  $8x + \tan x - 6 \ln |\sec x| + C$

(b)  $\frac{(3 - \tan x)^3}{3} + C$

(c)  $(3 - \tan x) \sec^2 x + C$

(d)  $9x + \frac{1}{3} \tan^3 x - 3 \tan^2 x + C$

(e)  $9x + \tan x - 6 \sec x + C$

$$= \int 9 - 6 \tan x + \sec^2 x - 1 dx$$

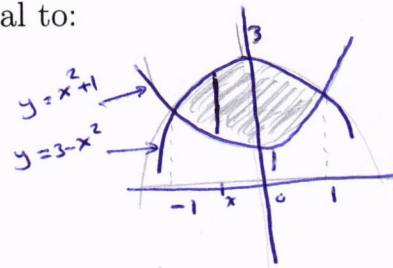
$$= \int 8 - 6 \tan x + \sec^2 x dx$$

$$= 8x - 6 \ln |\sec x| + \tan x + C$$

3. If  $f(x) = \int_3^{x^3} \sqrt[3]{t-t^2} dt$ , then  $f'(x) =$
- $$\begin{aligned} & \sqrt[3]{x^3 - (x^3)^2} \cdot \frac{d}{dx} [x^3] \\ &= \sqrt[3]{x^3 - x^6} \cdot 3x^2 \\ &= \sqrt[3]{x^3(1-x^3)} \cdot 3x^2 \\ &= x^3 \sqrt[3]{1-x^3} \cdot 3x^2 \\ &= 3x^3 \sqrt[3]{1-x^3} \end{aligned}$$
- (a)  $3x^3 \sqrt[3]{1-x^3}$
- (b)  $\sqrt[3]{x^3 - x^6}$
- (c)  $\sqrt[3]{x^3 - x^3} + \sqrt[3]{3}$
- (d)  $3x^2 \sqrt[3]{x - x^2}$
- (e)  $3x \sqrt[3]{1-x^3}$

4. The **area** of the region bounded by the curves  $y = x^2 + 1$  and  $y = 3 - x^2$  is equal to:

- (a)  $\frac{8}{3}$
- (b)  $\frac{4}{3}$
- (c)  $\frac{2}{5}$
- (d)  $\frac{1}{6}$
- (e)  $\frac{4}{5}$



pts of intersection:  
 $x^2 + 1 = 3 - x^2$   
 $\Rightarrow 2x^2 = 2 \Rightarrow x^2 = 1$   
 $\Rightarrow x = \pm 1$

$$\begin{aligned} A &= \int_{-1}^1 (3-x^2) - (x^2+1) dx \\ &= \int_{-1}^1 2 - 2x^2 dx \\ &= \left[ 2x - \frac{2}{3}x^3 \right]_{-1}^1 \\ &= \left( 2 - \frac{2}{3} \right) - \left( -2 + \frac{2}{3} \right) \\ &= 4 - \frac{4}{3} \\ &= \frac{12-4}{3} = \frac{8}{3} \end{aligned}$$