MATH 102.5 (151)

Quiz 1 (Sects 5.3, 5.4, 5.5)

Duration: 20 min

Name: ID:

1.) (4 pts) Use Riemann sum to write the integral $\int_{-1}^{0}[(x+1)^3+(x+1)^2]\,dx$ as limit of sums. Evaluate this limit.

2.) (6 pts) Evaluate the integrals

$$A=\int x^7\sqrt{x^4+1}\,dx \;;\; B=\int_1^2 (x^2-1)(x-1)^{29}dx \;;\; C=\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

1.) $$\int_{-1}^{0}[(x+1)^3+(x+1)^2]\,dx=\lim_{n\to\infty}\sum_{i=1}^{n}\Delta x\,[(x_i^*+1)^3+(x_i^*+1)^2]$$

$\Delta x=\frac{1}{n}$

$x_i^*=-1+\frac{i}{n}$

$$\Rightarrow \int_{-1}^{0}[(x+1)^3+(x+1)^2]\,dx=\lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}\left[\frac{i^3}{n^3}+\frac{i^2}{n^2}\right]\right]$$

$$=\lim_{n\to\infty}\left[\frac{1}{n}\left(\frac{1}{n^3}\frac{n^2(n+1)^2}{4}+\frac{1}{n^2}\frac{n(n+1)(2n+1)}{6}\right)\right]$$

$$=\frac{1}{4}+\frac{2}{6}=\frac{7}{12}$$

2.) $A=\int x^7\sqrt{x^4+1}\,dx$

$u=x^4+1$, $du=4x^3dx$

$x^4=u-1$

$$\Rightarrow A=\frac{1}{4}\int(u-1)\sqrt{u}\,du$$

$$=\frac{1}{4}\int\left(u^{3/2}-u^{1/2}\right)du$$

$$=\frac{1}{4}\left(\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}+C\right)$$

$$\boxed{A=\frac{1}{10}(x^4+1)^{5/2}-\frac{1}{6}(x^4+1)^{3/2}+C}$$

$B=\int_1^2(x^2-1)(x-1)^{29}\,dx$

$u=x-1$, $du=dx$

$x=u+1$

$$\Rightarrow B=\int_0^1(u+2)u^{30}\,du$$

$$=\int_0^1\left(u^{31}+2u^{30}\right)du$$

$$=\left[\frac{u^{32}}{32}+\frac{2u^{31}}{31}\right]_0^1$$

$$\boxed{B=\frac{1}{32}+\frac{2}{31}}$$

$C=\int\frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$u=\sqrt{x}$ $\quad du=\frac{1}{2\sqrt{x}}\,dx$

$C=\int 2e^u\,du$

$=2e^u+C$

$$\boxed{C=2e^{\sqrt{x}}+C}$$