MATH 102.5 (Term 151)

Quiz 5 (Sects. 10.1, 10.2 & 10.3) Duration: 20mn

Name: **ID number:**

1.)(2pts)Find the limit of the sequence $\{n-\sqrt{n^2+n+1}\}_{n=1}^{\infty}$.

2.)(4pts)What is the value of sum $\sum_{n=1}^{\infty}\frac{(2)^{1+n}}{(\sqrt{3})^{2+2n}}$, $\sum_{n=2}^{\infty}\frac{2}{n^2-1}$.

3.)(4pts)Do the following series converge or diverge $\sum_{n=1}^{\infty}\frac{n}{n^4+1}$, $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^4}$

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1) $u_n = n-\sqrt{n^2+n+1}$

$= \frac{n-(n^2+n+1)}{n+\sqrt{n^2+n+1}} = \frac{-n-1}{n+\sqrt{n^2+n+1}}$

$\lim_{n\to\infty} u_n = -\frac{1}{2}$

2) $\sum_{n=1}^{\infty}\frac{2^{1+n}}{(\sqrt{3})^{2+2n}} = \sum_{n=1}^{\infty}\frac{2^{1+n}}{3^{1+n}} = \frac{4}{9}\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^{n-1}$

$= \frac{\frac{4}{9}}{1-\frac{2}{3}} = \frac{4}{3}.$

$\sum_{n=2}^{\infty}\frac{2}{n^2-1} = \sum_{n=2}^{\infty}\left(\frac{1}{n-1}-\frac{1}{n+1}\right)$

$S_n = \left(1-\cancel{\tfrac{1}{3}}\right)+\left(\tfrac{1}{2}-\cancel{\tfrac{1}{4}}\right)+\left(\cancel{\tfrac{1}{3}}-\tfrac{1}{5}\right)+\cdots+\left(\tfrac{1}{n-2}-\tfrac{1}{n}\right)+\left(\cancel{\tfrac{1}{n-1}}-\tfrac{1}{n+1}\right)$

$S_n = 1+\frac{1}{2}-\frac{1}{n}-\frac{1}{n+1}$

$\lim_{n\to\infty} S_n = \frac{3}{2} \Rightarrow \sum_{n=2}^{\infty}\frac{2}{n^2-1} = \frac{3}{2}$

3.) We use integral test for both series. The functions $f(x)=\frac{x}{x^4+1}$ and $g(x)=\frac{1}{x(\ln x)^4}$ are positive, continuous and decreasing.

$I = \int_1^{\infty}\frac{x}{x^4+1}\,dx.$ | $u=x^2$, $du=2x\,dx$.

$I = \frac{1}{2}\int_1^{\infty}\frac{du}{u^2+1} = \frac{1}{2}\lim_{b\to\infty}\left[\tan^{-1}b-\tan^{-1}1\right]$

$= \frac{1}{2}\left(\frac{\pi}{2}-\frac{\pi}{4}\right) = \pi/8$

$\Rightarrow \sum_{n=1}^{\infty}\frac{n}{n^4+1}$ Converges.

$J = \int_2^{\infty}\frac{dx}{x(\ln x)^4}$ | $u=\ln x$, $du=\frac{dx}{x}$

$J = \int_{\ln 2}^{\infty}\frac{du}{u^4} = \lim_{b\to\infty}\left[\frac{-1}{3b}+\frac{1}{3(\ln 2)^3}\right]$

$= \frac{1}{3(\ln 2)^3}$

$\Rightarrow \sum_{n=2}^{\infty}\frac{1}{n(\ln n)^4}$ Converges.