MATH 102.5 (Term 151)

Quiz 6 (Sects. 10.4, 10.5 & 10.6) Duration: 30mn

Name: ID number:

1.)(2pts)Use comparison test to show that the series $\sum_{n=0}^{\infty} \frac{\sin^4\left(\frac{n^2}{n^3+1}\right)}{(n+2)^{4/3}}$ converges.

2.)(2pts)Use ratio test to study the convergence of the series $\sum_{n=1}^{\infty} \frac{(2n+1)!}{(n+1)^{2n}}$.

3.)(4pts)Do the series $\sum_{n=1}^{\infty} \left(\frac{2+\ln n}{2+n^2}\right)^n$ and $\sum_{n=1}^{\infty} \left(\frac{1+2n^3}{n^3+1}\right)^n$ converge or diverge?

4.)(2pts)Find the smallest number of terms required to approximate the sum $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^4}$ so that $|\text{error}| \leq 0.0001$.

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1) $\sin^4\left(\frac{n^2}{n^3+1}\right) \leq 1$

$$\frac{\sin^4\left(\frac{n^2}{n^2+1}\right)}{(n+2)^{4/3}} \leq \frac{1}{(n+2)^{4/3}}$$

$\sum \frac{1}{(n+2)^{4/3}}$ CV $\Rightarrow \sum \frac{\sin^4(n^2/n^2+1)}{(n+2)^{4/3}}$ CV

2) $$\frac{a_{n+1}}{a_n} = \frac{(2n+3)!}{(n+2)^{2n+2}} \frac{(n+1)^{2n}}{(2n+1)!}$$

$$= \frac{(2n+3)(2n+2)}{(n+2)^2}\left(\frac{n+1}{n+2}\right)^{2n}$$

$$\left(\frac{n+1}{n+2}\right)^{2n} = e^{2n\ln\left(\frac{n+1}{n+2}\right)} = e^{2\frac{\ln(n+1)-\ln(n+2)}{\frac{1}{n}}}$$

$$\xrightarrow[H.R]{n\to\infty} e^{\frac{2\left(\frac{1}{n+1}-\frac{1}{n+2}\right)}{-\frac{1}{n^2}}}$$

$$\xrightarrow{n\to\infty} e^{-2}$$

Thus, $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \frac{4}{e^2} < 1$

$\Rightarrow \sum \frac{(2n+1)!}{(n+1)^{2n}}$ CV

3) $$\sqrt[n]{a_n} = \frac{2+\ln n}{2+n^2} \xrightarrow{n\to\infty} 0 < 1$$

$\Rightarrow \sum \left(\frac{2+\ln n}{2+n^2}\right)^n$ CV

$$\sqrt[n]{a_n} = \frac{1+2n^3}{n^3+1} \xrightarrow{n\to\infty} 2 > 1$$

$\Rightarrow \sum \left(\frac{1+2n^3}{n^3+1}\right)^n$ Div

4) $|S-S_n| \leq a_{n+1} = \frac{1}{(n+1)^4}$

We solve $\frac{1}{(n+1)^4} \leq 0.0001 = 10^{-4}$

$\Rightarrow (n+1)^4 \geq 10^4$

$n+1 \geq 10$

$n \geq 9$

$n = 9$