Instructions:

1. Write clearly and legibly. You may lose points for messy work.

2. **Show all your work.** No points for answers without justifications.

3. **Calculators and Mobiles are not allowed.**

4. Make sure that you have 7 different problems (7 pages + cover page)

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<th>Question #</th>
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<th>Maximum Points</th>
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1. Given that \( w = \ln x \) is a solution of the differential equation:

\[
x^2 y'' + xy' + \frac{1}{\ln x} y = 1, \quad x > 1,
\]

find a particular solution for the differential equation

\[
x^2 y'' + xy' + \frac{1}{\ln x} y = 2 + \frac{1}{\ln x}, \quad x > 1.
\]
2. (a) Given that \( y_1(t) = t \) is a solution of the differential equation:

\[
t^2 y'' - t(t + 2) y' + (t + 2) y = 0, \quad t > 0.
\]

Reduce the given differential equation into a first order differential equation. (Do not solve the obtained equation !)

(b) Given that \( y_1 = x \cos(\ln x) \) is a solution of the differential equation:

\[
x^2 y'' - x y' + 2 y = 0 \quad \text{on } (0, \infty),
\]

find a second solution \( y_2 \) that is linearly independent of \( y_1 \).
3. Determine the form of a particular solution for the differential equation:

\[ y^{(4)} - 4y'' = 5x^2 - e^{2x}. \]
4. Find the general solution of the differential equation:

\[ y'' - y = x e^{-x} (x + \sin^2 x) - 3x + 1, \]

by the undetermined coefficients method. (Do not calculate the constant coefficients of the particular solution!)


5. Solve the differential equation:

\[ y'' + 2y' + y = e^{-x} \sec^2 x. \]
6. Solve the differential equation

\[ y'' + \frac{1}{x^2} y = 0, \quad x > 0. \]
7. Find a homogeneous **Cauchy-Euler equation** whose solution is given by:

\[ y = c_1 x^{-2} + c_2 \cos(2 \ln x) + c_3 \sin(2 \ln x) \]