151- MATH 301. Methods of Applied Mathematics
Quiz 4

1. Use separation of variables method to solve the problem:

\[ \frac{\partial u}{\partial t} = 3 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0 \]

subject to the boundary and initial conditions:

\[ \frac{\partial u}{\partial x}\bigg|_{x=0} = 0, \quad \frac{\partial u}{\partial x}\bigg|_{x=\pi} = 0, \quad t > 0 \]

\[ u(x, 0) = x, \quad 0 < x < \pi \]

2. Use separation of variables method to solve the problem:

\[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0, \quad 0 < r < 4, \quad 0 < z < 2 \]

subject to the boundary and initial conditions:

\[ u(4, z) = 0, \quad 0 < z < 2 \]
\[ u(r, 0) = 0, \quad 0 < r < 4 \]
\[ u(r, 2) = 4, \quad 0 < r < 4 \]

Also the solution is bounded at \( r = 4 \).

3. Find the steady state temperature \( T(r, \theta) \) in a sphere of radius 5 by solving the problem:

\[ \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial T}{\partial \theta} = 0, \quad 0 < r < 4, \quad 0 < \theta < \pi \]

subject to the boundary condition:

\[ T(5, \theta) = 1 + \cos 2\theta, \quad 0 < \theta < \pi \]

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