1. State and prove the "Sandwitch Theorem" for sequences.

2. If \( x > 0 \), prove that \( x^{n+1} + \frac{1}{x^{n+1}} > x^n + \frac{1}{x^n} \) for any \( n \in \mathbb{N} \).

3. Let \( S \) be a nonempty subset of real numbers, and let \( w = \inf S \).
   Prove that there exists a sequence \( (s_n)_{n \in \mathbb{N}} \) of elements of \( S \) such that
   \[
   w = \lim_{n \to +\infty} s_n.
   
   \]
   This sequence is called a minimizing sequence.

4. Assume that \( x_1 \) and \( x_2 \) are arbitrary real numbers with \( x_1 < x_2 \).
   Show that the sequence \( (x_n)_{n \in \mathbb{N}} \), defined by \( x_{n+1} = \frac{1}{2} (x_{n-1} + x_{n-2}) \)
   for \( n > 2 \), is convergent and find its limit.

5. Let \( (A_n)_{n \in \mathbb{N}} \) be a sequence of nonempty subsets of \( \mathbb{R} \) such that
   (i) \( A_1 \supset A_2 \supset A_3 \supset \ldots \), and
   (ii) \( |x - y| \leq \frac{1}{n} \) for all \( x, y \in A_n \).
   Let \( (a_n)_{n \in \mathbb{N}} \) be a sequence in \( \mathbb{R} \) such that \( a_n \in A_n \) for each \( n \).
   Prove that \( (a_n)_{n \in \mathbb{N}} \) is convergent.