1. If $x > 0$ show that
   \[
   \left| (1 + x)^{1/3} - (1 + \frac{x}{3} - \frac{x^2}{9}) \right| \leq \frac{5x^3}{81}.
   \]
   Use this inequality to approximate $\sqrt[3]{2}$.

2. Let $I$ be an open interval in $\mathbb{R}$, let $f : I \to \mathbb{R}$ be differentiable on $I,$ and suppose $f''(a)$ exists at $a \in I.$ Show that
   \[
   f''(a) = \lim_{h \to 0} \frac{f(a + h) - 2f(a) + f(a - h)}{h^2}.
   \]

3. Show that if $a > 0,$ then the sequence $(f_n)$ defined by $f_n(x) = n^2x^2e^{-nx},$ $x \in \mathbb{R},$ converges uniformly on $[a, +\infty),$ but it does not converge uniformly on $[0, +\infty).$

4. Discuss the convergence and uniform convergence of the series
   \[
   \sum_{n=0}^{\infty} \frac{1}{x^2 + n^2} \text{ on } \mathbb{R}.
   \]