Problem 1 (21 points):

(a) Define Hamiltonian graph.
(b) Give five conditions on the graph $G$ (of order $n \geq 2$), each of which implies that $G$ is Hamiltonian.
(c) Determine the connectivity and the edge-connectivity of the graph in the picture.
(d) Is the graph Eulerian? Why?
(e) Is the graph Hamiltonian? Why?

Problem 2 (15 points):

(a) State Menger’s Theorem.
(b) Show that $\kappa(Q_n) = \lambda(Q_n) = n$ for $n \geq 2$.

Problem 3 (36 points): Determine whether each of the following statements is true or false. If a statement is true sketch the proof, and if it is false, give a counter example.

(a) If $G_1$ and $G_2$ are Eulerian, then $G_1 \vee G_2$ is Eulerian.
(b) If a graph $G$ is pancyclic, then $G$ is panconnected.
(c) Let $G$ be nontrivial graph and $v \in V(G)$. Then $\kappa(G-v) = \kappa(G)$ or $\kappa(G-v) = \kappa(G) - 1$.
(d) Assume that $G = K_{m,n}$ ($m \geq n \geq 2$) has odd degree, then $G$ is Hamiltonian.
(e) If $G$ is Eulerian, then $L(G)$ is Eulerian too.
(f) No bipartite graph is Hamiltonian-connected.

Problem 4 (28 points): Short proofs. Prove each of the following

1) If $T$ is a tree containing at least one vertex of degree 2, then $\bar{T}$ is not Eulerian.
2) If $d(v) \geq \frac{n}{2}$ for each vertex $v$ of a graph $G$ of order $n \geq 3$, then $k(G - S) \leq |S|$ for each nonempty proper subset $S$ of $V(G)$.
3) Every 1-tough graph is 2-connected.
4) If $G$ is nontrivial connected graph, then $T(G^2)$ is Hamiltonian.