Problem 1 (25 points):

(a) Give an example of a planar graph which isomorphic to its dual graph.

(b) Find the number of distinct labeling of the graph in the figure.

(c) Give a maximal planar graph of order 6
Problem 3 (25 points): Either prove or disprove each of the following statements. If a statement is true sketch the proof, and if it is false, give a counter example.

(a) Every induced subgraph of the complete graph $K_n$ is complete.

$G = K_4 - e$

(b) If $k$ is an odd integer and $G$ is a $k$-regular graph of size $m$, then $m$ is a multiple of $k$.

(c) If $G_1$ and $G_2$ are regular graphs, then the join $G_1 \lor G_2$ is regular.

(d) If the graph $G$ has only two vertices of odd degree, then they must be connected by a path.

(e) Any connected graph has only one central vertex.
Problem 4 (29 points):

1) Let $G$ is a graph of order $2n$ and size $m$. If $\delta(G) \geq n$ for each vertex $v$, prove that $G$ is connected.

2) Prove that if $G$ is an acyclic graph of order $n$ and size $m$ such that $m = n - 1$, then $G$ is a tree.
3) Let $G$ be a connected graph of order $n$ ($n \geq 3$). Prove that there is an orientation of $G$ in which no directed path has length 2 if and only if $G$ is bipartite.

4) Apply Kruskal’s algorithm to find a minimum spanning tree $T$ in the weighted graph $G$. Show how this tree is constructed. Also find $w(T)$. 

![Graph Diagram]