(1)(12 points) Old hens can be bought at SAR 2 each and young ones at SAR 5 each. The old hens lay 3 eggs per week and young ones lay 5 eggs per week, each eggs being worth 30 halala. A hen costs SAR 1 per week to feed. I have only SAR 80 to spend for hens, how many of each kind should I buy to give a profit of more than SAR 6 per week, assuming that I cannot house more than 20 hours.

(2a)(08 points) Define basic feasible solutions of a linear programming problem. Find all basic feasible solutions for the system of simultaneous equations.
\[ \begin{align*}
    x_1 - x_2 + 2x_3 + 2x_4 &= 4 \\
    3x_1 + 4x_3 + 6x_4 &= 1.
\end{align*} \]

(2b)(08 points) Show that feasible region of every linear programming problem is convex set.

(3)(12 points) Solve by Simplex Method
\[ \text{Maximize } z = -x_1 + 3x_2 - 2x_3 \]
subject to
\[ \begin{align*}
    3x_1 - x_2 + 2x_3 &\leq 7 \\
    -2x_1 + 4x_2 &\leq 12 \\
    -4x_1 + 3x_2 + 8x_3 &\leq 10 \\
    x_1, x_2, x_3 &\geq 0.
\end{align*} \]

(4)(08 points) Write the dual of the following linear programming problem
\[ \text{Maximize } z = 3x_1 + x_2 + 2x_3 - x_4 \]
subject to
\[ \begin{align*}
    2x_1 - x_2 + 3x_3 + x_4 &= 1 \\
    x_1 + x_2 - x_3 + x_4 &= 3 \\
    x_1, x_2, x_3 &\geq 0, \quad x_4 \text{ is unrestricted.}
\end{align*} \]

(5)(12 points) What are artificial variables? Why do we need them? Use two-phase simplex method to solve:
\[ \text{Maximize } z = -x_1 - x_2 \]
subject to
\[ \begin{align*}
    2x_1 + x_2 &\geq 4 \\
    x_1 + 7x_2 &\geq 7 \\
    x_1, x_2 &\geq 0.
\end{align*} \]