

Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS475: Survival Models for Actuaries
Dr. Mohammad H. Omar
Major 1 Exam Term 152 FORM A **SOLUTION**
Monday Feb 22 2016
6.00pm-7.30pm

Name _____ ID#: _____ Serial #: _____

Instructions.

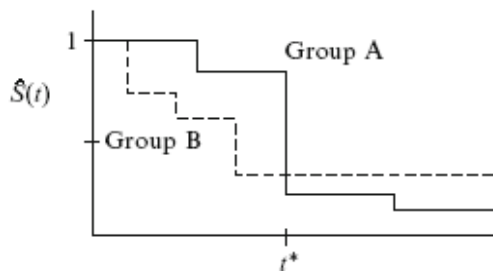
1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	1+1+1=3		
2	4		
3	10+3+7=20		
4	3+3+3+5=14		
5	3+5+6=14		
6	4+1=5		
Total	60		

Extra blank page

1. (1+1+1=3 points) Consider the comparison of the following two survivor curves in the graph below:



- (a) Which group has a better survival prognosis (chance) before time t^* ?
- (b) Which group has a better survival prognosis after time t^* ?
- (c) Which group has a longer median survival time?

2. (4 points) Forty losses have been observed.

Loss amount X (in millions)	Number of losses	Sum of loss amount (in millions)
$(1, 4/3]$	16	20
$(4/3, 2]$	10	15
$(2, 4]$	10	35
more than 4	4	

Using the empirical model based on these observations, **determine** $E(X \wedge 2)$, where $X \wedge 2$ is $\min(X, 2)$.

3. (10+3+7=20 points) The following data are a sample from the 1967-1980 Evans County study. Survival times (in years) are given for two study groups, each with 25 participants. Group 1 has no history of chronic disease (CHR = 0), and Group 2 had a positive history of chronic disease (CHR = 1):

Group 1 (CHR = 0): 12.3+, 5.4, 8.2, 12.2+, 11.7, 10.0, 5.7, 9.8, 2.6, 11.0, 9.2, 12.1+,
6.6, 2.2, 1.8, 10.2, 10.7, 11.1, 5.3, 3.5, 9.2, 2.5, 8.7, 3.8, 3.0

Group 2 (CHR = 1): 5.8, 2.9, 8.4, 8.3, 9.1, 4.2, 4.1, 1.8, 3.1, 11.4, 2.4, 1.4, 5.9, 1.6, 2.8,
4.9, 3.5, 6.5, 9.9, 3.6, 5.2, 8.8, 7.8, 4.7, 3.9

- (a) Fill in the missing information in the following table of ordered failure times for groups 1 and 2.

Group 1					Group 2				
$t_{(f)}$	n_f	m_f	q_f	$S(t_{(f)})$	$t_{(f)}$	n_f	m_f	q_f	$S(t_{(f)})$
0.0	25	0	0	1.00	0.0	25	0	0	1.00
1.8	25	1	0	0.96	1.4	25	1	0	0.96
2.2	24	1	0	0.92	1.6	24	1	0	0.92
2.5	23	1	0	0.88	1.8	23	1	0	0.88
2.6	22	1	0	0.84	2.4	22	1	0	0.84
3.0	21	1	0	0.80	2.8	21	1	0	0.80
3.5	20		0		2.9	20	1	0	0.76
3.8	19	1	0	0.72	3.1	19	1	0	0.72
5.3	18	1	0	0.68	3.5	18	1	0	0.68
5.4	17	1	0	0.64	3.6	17	1	0	0.64
5.7	16	1	0	0.60	3.9				
6.6	15	1	0	0.56	4.1			0	
8.2	14	1	0	0.52	4.2	14	1	0	0.52
8.7	13	1	0	0.48	4.7	13	1	0	0.48
9.2			0		4.9	12	1	0	0.44
9.8	10	1	0	0.36	5.2	11	1	0	0.40
10.0	9	1	0	0.32	5.8	10	1	0	0.36
10.2	8	1	0	0.28	5.9	9	1	0	0.32
10.7	8	1	0	0.24	6.5	8	1	0	0.28
11.0	6	1	0	0.20	7.8	7	1	0	0.24
11.1	5	1	0	0.16	8.3	6	1	0	0.20
11.7	4	1			8.4	5	1	0	0.16
					8.8	4	1	0	0.12
					9.1			0	
					9.9			0	
					11.4	1	1	0	0.00

- (b) The following partial results were obtained for the data. Complete the missing information.

Group	Events Observed	Events expected
1		30.79
2	25	
Total	47	47.00

- (c) You are also given that $Var(O_i - E_i) = 9.658$.

i) Together with results in part b, **compute** the log-rank statistic.

ii) Use this statistic to carry out the **log-rank test** for these data.

iii) What is your **null hypothesis** and how is the **test statistic distributed** under this null hypothesis?

iv) What are your **conclusions** from the test?.

4. (3+3+3+5=14 points) Consider the following situations.

a) Ten insurance payments were recorded as follows :

4, 4, *5*, *5*, *5*, 8, *10*, *10*, 12, and 15,

with the *italicised* values representing payments at a **policy limit**. There were no deductibles.

Determine the **product limit estimate** of $S(11)$ and **Greenwood's approximation** of its variance.

b) All observations begin on day zero. Eight observations were

4+, 8+, 8, 12, 12+, 12, 22, and 36,

with "+" values representing right censored observations.

Determine the **Nelson Aalen estimate** of $H(12)$ and then determine a **90% linear confidence interval** for the true value.

5. (3+5+6=14 points) Given five observations

82, 126, 161, 294, and 384,

determine each of the following:

- (a) The **empirical estimate** of $F(150)$.
- (b) The kernel density estimate of $F(150)$ based on a **uniform kernel** with bandwidth $b = 50$.
- (c) The kernel density estimate of $F(150)$ based on a **triangular kernel** with bandwidth $b = 50$.

6. (4+1=5 points) For 200 auto accident claims you are given:

(i) Claims are submitted t months after the accident occurs, 0,1, 2...

(ii) There are no censored observations.

(iii) $\hat{S}(t)$ is calculated using the Kaplan-Meier product limit estimator.

(iv) $c_s^2(t) = \frac{\widehat{Var}[\hat{S}(t)]}{\hat{S}(t)^2}$, where $\widehat{Var}[\hat{S}(t)]$ is calculated using Greenwood's approximation.

(v) $\hat{S}(8) = 0.22$, $\hat{S}(9) = 0.16$, $c_s^2(9) = 0.02625$, $c_s^2(10) = 0.04045$

Determine the number of claims that were submitted to the company 10 months after an accident occurred.

(A) 10

(B) 12

(C) 15

(D) 17

(E) 18

Work Shown (4 marks) and Final answer (1 mark)

END OF TEST PAPER