1. The sum of the critical numbers of the function \( f(x) = \frac{x - 1}{x^2 + 4} \) is

a) 2 
b) 3 
c) 0 
d) 4 
e) 5

2. If \( 2x + 1 \leq h(x) \leq x^4 - x^2 + 3 \) for all \( x \in (-2, 2) \), then \( \lim_{x \to 1} h(x) = \)

a) 3 
b) 2 
c) 4 
d) 0 
e) Does not exist
3. The largest value for $\delta$, such that if $|x - 2| < \delta$ then $|4x - 8| < 0.01$, is

a) 0.0025  
b) 0.025  
c) 0.25  
d) 0.01  
e) 0.0001

4. Let $f(x) = \begin{cases} 
x^2 - a, & x \leq a \\
2, & a < x < b \\
3x + b & x \geq b
\end{cases}$ be a continuous function everywhere with $a \neq b$.
Then $a + b =$

a) 1  
b) $\frac{1}{4}$  
c) 2  
d) $\frac{3}{4}$  
e) 0
5. If \( f(x) = \frac{x^2 - 1}{x^2 + 1} \), then \( f'(1) = \\

a) 1 \\
b) \frac{1}{4} \\
c) 4 \\
d) 2 \\
e) \frac{1}{2} \\

6. A particle moves according to the law of motion \( s(t) = \cos\left(\frac{\pi t}{4}\right) \) where \( s \) is measured in meters and \( t \) is measured in seconds. The total distance travelled during the first 8 seconds is

a) 4 m \\
b) 2 m \\
c) 8 m \\
d) 6 m \\
e) 10 m
7. Which one of the following statements is TRUE about \( f(x) = \begin{cases} 
\ x + 2 & \text{if } x < 0 \\
\ e^x & \text{if } 0 \leq x \leq 1 \\
\ 2 - x & \text{if } x > 1 
\end{cases} \)?

a) \( f(x) \) is continuous from the left at \( x = 1 \)
b) \( f(x) \) is continuous at \( x = 0 \)
c) \( f(x) \) is differentiable at \( x = 1 \)
d) \( f(x) \) is continuous from the left at \( x = 0 \)
e) \( f(x) \) is continuous from the right at \( x = 1 \)

8. Suppose that \( a^{x^2}y + x\sqrt{y} = a + 1 \) where \( a > 0 \). Then \( \frac{dy}{dx} \) at the point \((1, 1)\) is equal to

a) None of the above (this answer was the last choice)
b) \( \ln a \)
c) \( a \)
d) \( 5 \)
e) \( -2 \)
9. If \( f(1) = -1 \) and \( f'(x) \leq 2 \) for all \( x \). The largest possible value of \( f(2) \) is

a) 1  
b) 0  
c) 2  
d) -1  
e) -2

10. The \( x \)-intercept of the tangent line to the curve \( y = x + \tan x \) at the point \((\pi, \pi)\) is

a) \( \pi/2 \)  
b) \( \pi/4 \)  
c) \( 2\pi/3 \)  
d) \( -\pi \)  
e) \( \pi/6 \)
11. \( \lim_{{x \to 0^+}} \left[ \frac{2}{\pi} \tan^{-1}(\ln x) \right] = \)

a) \(-1\)  
b) \(\frac{\pi}{2}\)  
c) 0  
d) \(\infty\)  
e) \(-\infty\)

12. \( \lim_{{x \to \infty}} \left( \sqrt{x^2 + ax} - \sqrt{x^2 + bx} \right) = \)

a) \(\frac{a - b}{2}\)  
b) \(a - b\)  
c) \(\frac{a + b}{2}\)  
d) \(a + b\)  
e) 0
13. If \( f''(x) = x^{-2}, \ x > 0, \ f(1) = 0, \) and \( f(2) = 0, \) then

a) \( f(x) = -\ln x + (\ln 2) x - \ln 2 \)
b) \( f(x) = \frac{-1}{x} \)
c) \( f(x) = \frac{1}{x} \)
d) \( f(x) = -\ln x - (\ln 2)x + \ln 2 \)
e) \( f(x) = -\ln x - (\ln 2)x - \ln 2 \)

14. \( \lim_{x \to 1} \left[ \frac{\ln(x^3)}{2x-2} \right] = \)

a) \( \frac{3}{2} \)
b) \( \frac{1}{2} \)
c) \( \frac{1}{3} \)
d) \( \frac{2}{3} \)
e) 1
15. If \( f'(x) = (x^2 + x + 1)^4 (x - 3)^3 (x - 1)(x^2 - x + 1)^6 \), then \( f \) is decreasing on

a) \((1, 3)\)
b) \((-\infty, 1)\)
c) \((3, \infty)\)
d) \((-\infty, 1) \cup (3, \infty)\)
e) \((-\infty, \infty)\)

16. If \( f' \) is continuous everywhere, \( f(7) = 0 \) and \( f'(7) = 2 \), then

\[
\lim_{x \to 0} \left[ \frac{f(7 + 5x) + f(7 + 8x)}{x} \right] =
\]

a) 26
b) 40
c) 49
d) 14
e) 10
17. If \( x_1 = 0 \) is an approximation to one root of the equation \( x^3 = 1 - x \), then the approximation \( x_3 \) given by Newton's Method is

a) \( \frac{3}{4} \)

b) \( \frac{1}{2} \)

c) \( \frac{1}{4} \)

d) \( \frac{3}{2} \)

e) \( \frac{2}{5} \)

18. \( \lim_{x \to 1^+} [\ln(x^\pi - 1) - \ln(x^e - 1)] = \)

a) \( (\ln \pi) - 1 \)

b) 0

c) \( +\infty \)

d) \( \ln (\pi - e) \)

e) 1
19. If \( \tanh x = \frac{12}{13} \), then \( 5 \sinh x + 13 \sech x = \)

a) 17  
b) 18  
c) 60  
d) 25  
e) 22

20. If \( \lim_{x \to 0} \left( \frac{\sqrt{Mx + N} - 2}{x} \right) = 1 \) then

a) \( M = N \)  
b) \( M \neq N \)  
c) \( M > N \)  
d) \( M < N \)  
e) \( M \) and \( N \) cannot be found
21. If \( y = ax + b \) and \( y = cx + d \) are equations of lines that are tangent to the curve \( y = 1 + x^3 \) and parallel to the line \( 12x - y = 1 \), then \( b + d = \)

a) 2  
b) 4  
c) 6  
d) 12  
e) 0

22. Let \( g(x) = x^{1/3}(x + 4) \). Which one of the following statements is TRUE?

a) \( g \) is concave downward on \( (0, 2) \) 
b) \( g \) is concave upwardward on \( (-1, \infty) \) 
c) \( g \) is concave downward on \( (-\infty, 0) \) 
d) \( g \) is concave upward on \( (0, \infty) \) 
e) \( g \) is concave downward on \( (2, 4) \)
23. The point, at which the slant asymptote and the vertical asymptote of the graph of 
\( f(x) = \frac{3x^2 + x - 2}{2x + 6} \) intersect, is 

a) \((-3, -\frac{17}{2})\)

b) \((-3, \frac{3}{2})\)

c) \((-3, -4)\)

d) \((0, 0)\)

e) \((3, \frac{17}{2})\)

24. Suppose \( f(x) = x^a (1 - x)^b \), where \(0 \leq x \leq 1\) and both of \(a\) and \(b\) are positive numbers. The maximum value of \(f\) equals to

a) \(\frac{a^ab^b}{(a + b)^{a+b}}\)

b) \(\frac{a}{a + b}\)

c) 1

d) \(\frac{a + b}{a^a + b}\)

e) \(\frac{b^b}{(a + 2b)^b}\)
25. The surface area of a sphere was found $36\pi \text{ cm}^2$ with possible error in the measurement of its radius 0.1 cm. Using differentials, the maximum possible error in computing its surface area is (Hint: Surface area of sphere is $4\pi r^2$)

   a) $\frac{12\pi}{5}$  
   b) $\frac{9\pi}{25}$  
   c) $\frac{7\pi}{5}$  
   d) $\frac{5\pi}{36}$  
   e) $\frac{6\pi}{7}$

26. Which one of the following statements is TRUE

   a) If $f'(x)$ exists at $x = r$, then $\lim_{{x \to r}} f(x) = f(r)$
   b) If $|f|$ is a continuous function at $a$, then $f$ is a continuous function at $a$
   c) $\lim_{{x \to 4}} \left( \frac{2x}{x-4} - \frac{8}{x-4} \right) = \lim_{{x \to 4}} \left( \frac{2x}{x-4} \right) - \lim_{{x \to 4}} \left( \frac{8}{x-4} \right)$
   d) If $\lim_{{x \to 5}} f(x) = 0$ and $\lim_{{x \to 5}} g(x) = 0$, then $\lim_{{x \to 5}} \left( \frac{f(x)}{g(x)} \right)$ doesn’t exist
   e) If $f(1) > 0$ and $f(3) < 0$ then there exists a number $c \in (1, 3)$ such that $f(c) = 0$
27. The area of the largest rectangle inscribed in a semi-circle with radius \( r \) is

a) \( r^2 \)
b) \( \frac{r^2}{2} \)
c) \( \frac{3r^2}{4} \)
d) \( \frac{\pi r^2}{2} \)
e) \( r \)

28. The values of the constants \( a, b, c \) and \( d \), so that \( f(x) = ax^3 + bx^2 + cx + d \) has a local maximum at the point \((0, 0)\) and a local minimum at the point \((1, -1)\), are as follows

a) \( a = 2, b = -3, c = 0, \) and \( d = 0 \)
b) \( a = -2, b = 1, c = 1, \) and \( d = 0 \)
c) \( a = 1, b = -1, c = 2, \) and \( d = 0 \)
d) \( a = 0, b = 2, c = -1, \) and \( d = 0 \)
e) \( a = -2, b = 0, c = 0, \) and \( d = 1 \)