1. Sketch the graph of an example of a function $f$ that satisfies all of the given conditions. 

$f(0) = 3, \lim_{x \to 0} f(x) = 4, \lim_{x \to 0^+} f(x) = 2,$

$\lim_{x \to -\infty} f(x) = -\infty, \lim_{x \to +\infty} f(x) = -\infty, \lim_{x \to -\infty} f(x) = \infty, \lim_{x \to +\infty} f(x) = 3$

2. Use the definition of a derivative to find $f'(x)$ and $f''(x)$. Then graph $f, f', \text{ and } f''$ on a common screen and check to see if your answers are reasonable.

$f(x) = 3x^3 + 2x + 1$

$$g(x) = f'(x) = \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + 2x + 2h + 1 - 3x^2 - 2x - 1}{h} = \lim_{h \to 0} \frac{6x + h + 2}{h}$$

$$f''(x) = g'(x) = \lim_{h \to 0} \frac{6(x + h) + 2 - 6x - 2}{h} = \lim_{h \to 0} \frac{6h}{h} = 6$$

$f\left(-\frac{1}{3}\right) = \frac{2}{3}$

(Clearly the slope of $f(x)$ increases from $-\infty$ to zero at $-\frac{1}{3}$ then to $\infty$. And the slope of $f'(x)$ is fixed to be 6 because it is a straight line.)