MATH 202.5 (Term 152)

Quiz 1 (Sects. 1.2, 2.2 & 2.3) **Duration: 20mn**

Name: **ID number:**

1.)(5pts)Solve the IVP: $\begin{cases} \frac{dy}{dx} = y(y+1)\sin^2 x \\ y(0) = 1. \end{cases}$

2.)(5pts)Solve the DE: $x\frac{dy}{dx} - y = \frac{x^2}{x^2+1}$.

1.) The trivial solutions are $y=0$ and $y=-1$

But, both solutions are rejected because they don't satisfy $y(0)=2$.

Now, we look for non trivial solutions.

We assume $y \neq 0, -1$

The DE is seperable.

$$\int \frac{dy}{y(y+1)} = \int \sin^2 x \, dx$$

$$\int \left(\frac{1}{y} - \frac{1}{y+1}\right) dy = \int \frac{1-\cos 2x}{2} dx$$

$$\ln\left|\frac{y}{y+1}\right| = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$y(0)=1 \Rightarrow C = \ln\left(\frac{1}{2}\right)$$

$$\frac{y}{y+1} = \frac{1}{2} e^{\frac{x}{2} - \frac{\sin 2x}{4}}$$

or, $\boxed{y = \dfrac{\frac{1}{2} e^{\frac{x}{2} - \frac{\sin 2x}{4}}}{1 - \frac{1}{2} e^{\frac{x}{2} - \frac{\sin 2x}{2}}}, \ x \in I}$

2.) We can't find trivial solutions

This is a linear DE

$$\frac{dy}{dx} - \frac{y}{x} = \frac{x}{x^2+1}$$

$$e^{-\int \frac{dx}{x}} = e^{-\ln|x|} = \frac{1}{x}, \ x>0, \text{ is}$$

an integrating factor.

Thus, we have

$$\frac{d}{dx}\left(\frac{y}{x}\right) = \frac{1}{x^2+1}$$

$$\frac{y}{x} = \int \frac{dx}{x^2+1}$$

$$= \tan^{-1} x + C$$

$$\boxed{y = x\left(\tan^{-1} x + C\right), \ x \in (0, \infty)}$$