MATH 202.5 (Term 152)

Quiz 2 (Sects. 2.4 & 2.5) Duration: 20min

Name: ID number:

1.)(5pts)Solve the DE: $ye^{-xy} + x\cos 2x + xe^{-xy}\frac{dy}{dx} = 0$.

2.)(5pts)Solve the DE: $(x+1)^2\frac{dy}{dx} - y = y^2$.

1) $\underbrace{(ye^{-xy} + x\cos 2x)}_{M} dx + \underbrace{xe^{-xy}}_{N} dy = 0$

$M_y = e^{-xy} - xy e^{-xy}$

$N_x = e^{-xy} - xy e^{-xy}$

$M_y = N_x \Rightarrow$ DE is exact.

$\begin{cases} \frac{\partial f}{\partial x} = ye^{-xy} + x\cos 2x & (1) \\ \frac{\partial f}{\partial y} = xe^{-xy} & (2) \end{cases}$

We integrate (2)

$f(x,y) = \int x e^{-xy} dy$

$= -e^{-xy} + g(x)$

We substitute into (1)

$ye^{-xy} + g'(x) = ye^{-xy} + x\cos 2x$

$g'(x) = x\cos 2x$

$g(x) = \int x\cos 2x\, dx$

$u = x \rightarrow u' = 1$

$v' = \cos 2x \rightarrow v = \frac{\sin 2x}{2}$

$\Rightarrow g(x) = \frac{x\sin 2x}{2} + \frac{\cos 2x}{4}$

Thus, $\boxed{-e^{-xy} + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4} = C}$

$\boxed{\text{or} \quad y = \frac{-\ln\left|C + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right|}{x}, x \in I}$

2.) $y = 0$ is a trivial solution.

Assume $y \neq 0$: $\frac{dy}{dx} - \frac{1}{(x+1)^2}y = \frac{y^2}{(x+1)^2}, \quad x \neq -1$

This is Bernoulli's DE

let $u = y^{1-2} = y^{-1}$

$\Rightarrow y = u^{-1}, \quad \frac{dy}{dx} = -u^{-2}\frac{du}{dx}$

$\Rightarrow -u^{-2}\frac{du}{dx} - \frac{u^{-1}}{(x+1)^2} = \frac{u^{-2}}{(x+1)^2}, \quad x \neq -1$

$\frac{du}{dx} + \frac{u}{(x+1)^2} = -\frac{1}{(x+1)^2}, \quad x \neq -1$

$e^{\int \frac{dx}{(x+1)^2}} = e^{-\frac{1}{x+1}}$ is an integrating factor

$\Rightarrow \frac{d}{dx}\left(u e^{-\frac{1}{x+1}}\right) = -\frac{e^{-\frac{1}{x+1}}}{(x+1)^2}$

$u e^{-\frac{1}{x+1}} = -\int \frac{e^{-\frac{1}{x+1}}}{(x+1)^2} dx$

$v = -\frac{1}{x+1}, \quad dv = \frac{1}{(x+1)^2}dx$

$= -\int e^v dv = -e^v + C$

Thus,

$u e^{-\frac{1}{x+1}} = -e^{-\frac{1}{x+1}} + C$

$u = -1 + Ce^{\frac{1}{x+1}}$

$\boxed{y = \frac{1}{-1 + Ce^{\frac{1}{x+1}}}, \quad x \in (-1, \infty)}$