

MATH 202.5 (Term 152)

Quiz 3 (Sects. 4.2, 4.3 & 4.5)

Duration: 20min

Name: _____

ID number: _____

1.) (3pts) Use reduction of order (only) to find a second solution y_2 of the DE: $4x^2y'' + 5y = 0$, giving that $y_1 = x^{1/2} \cos \ln x$ is a solution on $(0, \infty)$.

2.) (3pts) Solve the DE: $2y''' - y'' - 2y' + y = 0$.

3.) (4pts) Find an operator of lowest order that annihilates $f(x) = x^2 e^{-x} \sin 2x \cos x$
 Hint: $\sin(a+b) = \sin a \cos b + \cos a \sin b$.

1) $y' + \frac{5}{4x^2}y = 0, P(x) = 0$

$$y_2 = y_1 \int \frac{e^{-\int P(x) dx}}{y_1^2} dx$$

$$= y_1 \int \frac{1}{x \cos^2(\ln x)} dx$$

let $u = \ln x, du = \frac{dx}{x}$

$$\Rightarrow y_2 = y_1 \int \frac{1}{\cos^2 u} du$$

$$= y_1 \tan u = y_1 \frac{\sin(\ln x)}{\cos(\ln x)}$$

$$y_2 = x^{1/2} \sin \ln x$$

2.) Its auxiliary equation is

$$2m^3 - m^2 - 2m + 1 = 0$$

$m = 1$ is a root.

$$\begin{array}{r} 2m^3 - m^2 - 2m + 1 \\ - (2m^3 - 2m^2) \quad \underline{m-1} \\ \hline m^2 - 2m + 1 \\ - (m^2 - m) \\ \hline -m + 1 \\ - (-m + 1) \\ \hline 0 \end{array}$$

$$A = 1 + 8 = 9$$

$$m_1 = \frac{-1-3}{2} = -1, m_2 = \frac{-1+3}{2} = 1$$

$$\Rightarrow y = C_1 e^{-x} + C_2 e^x + C_3 e^{x/2}, x \in (-\infty, \infty)$$

3.) $\sin a \cos b = \frac{1}{2} [\sin(a+b) + \sin(a-b)]$

$$\Rightarrow \sin 2x \cos x = \frac{1}{2} [\sin 3x + \sin x]$$

$$\Rightarrow f(x) = x^2 \frac{e^{-x}}{2} \sin 3x + x^2 \frac{e^{-x}}{2} \sin x$$

$$\begin{array}{cc} \downarrow & \downarrow \\ (D^2 + 2D + 1)^3 & (D^2 + 2D + 2)^3 \end{array}$$

$$\Rightarrow (D^2 + 2D + 1)^3 (D^2 + 2D + 2)^3 [f(x)] = 0$$