

MATH 202.5 (Term 152)

Quiz 4 (Sects. 4.6 & 4.7)

Duration: 20min

Name: \_\_\_\_\_

ID number: \_\_\_\_\_

1.) (5pts) Solve the DE:  $y'' + 6y' + 8y = \cos e^{2x}$ .

2.) (5pts) Solve the DE:  $x^3 y''' + 3x^2 y'' + xy' + y = 0$ .

1) We solve  $y'' + 6y' + 8y = 0$   
Its auxiliary equation is  $m^2 + 6m + 8 = 0$

$$m = -4, m = -2$$

$$\Rightarrow y_c = C_1 e^{-4x} + C_2 e^{-2x}$$

We use variation of parameters to find  $y_p$ .

We have  $y_p = u_1 y_1 + u_2 y_2$ ,

$$u_1' = \frac{-y_2 f(x)}{W}, \quad u_2' = \frac{y_1 f(x)}{W},$$

with  $y_1 = e^{-4x}$ ,  $y_2 = e^{-2x}$ ,  $f(x) = \cos e^{2x}$

$$W = \begin{vmatrix} e^{-4x} & e^{-2x} \\ -4e^{-4x} & -2e^{-2x} \end{vmatrix} = 2e^{-6x}$$

$$u_1' = \frac{-e^{-2x} \cos e^{2x}}{2e^{-6x}} = -\frac{1}{2} e^{4x} \cos e^{2x}$$

$$\Rightarrow u_1 = -\frac{1}{2} \int e^{4x} \cos e^{2x} dx$$

$$w = e^{2x}, \quad dw = 2e^{2x} dx$$

$$\int e^{4x} \cos e^{2x} dx = \frac{1}{2} \int w \cos w dw$$

$$= \frac{1}{2} (w \sin w + \cos w)$$

$$\Rightarrow u_1 = \frac{1}{4} (e^{2x} \sin e^{2x} + \cos e^{2x})$$

$$u_2' = \frac{e^{-4x} \cos e^{2x}}{2e^{-6x}} = \frac{1}{2} e^{2x} \cos e^{2x}$$

$$u_2 = \frac{1}{2} \int e^{2x} \cos e^{2x} dx = \frac{1}{4} \sin e^{2x}$$

$$\Rightarrow y_p = -\frac{1}{4} e^{-4x} \cos e^{2x}$$

$$\text{and } \underline{y = y_c + y_p}$$

2)  $y = x^m$

The auxiliary equation of the DE

$$\Rightarrow m(m-1)(m-2) + 3m(m-1) + m + 1 = 0$$

$$m^3 + 1 = 0$$

$$(m+1)(m^2 - m + 1) = 0$$

$$m = -1, \quad \Delta = 1 - 4 = -3$$

$$m = \frac{1 \pm i\sqrt{3}}{2}$$

$$\Rightarrow y = C_1 x^{-1} + x^{\frac{1}{2}} \left( C_2 \cos\left(\frac{\sqrt{3}}{2} \ln x\right) + C_3 \sin\left(\frac{\sqrt{3}}{2} \ln x\right) \right)$$

$$x \in (0, \infty)$$