

MATH 202.5 (Term 152)

Quiz 5 (Sects. 6.2 & 6.3)

Duration: 30min

Name: _____

ID number: _____

- 1.) (5pts) Find 2 power series solutions of the DE: $(x-4)y'' - y' - 2y = 0$.
- 2.) (2pts) Find the indicial roots of the DE $x^2y'' + 3x(x^4-1)y' - (1-x^3)y = 0$ at $x=0$.
- 3.) (3pts) Find a relation of recurrence satisfied by c_n , where $y = \sum_{n=0}^{\infty} c_n x^{n+\frac{1}{2}}$ is solution of the DE $2xy'' + y' + 2y = 0$.

1) $y = \sum_{n=0}^{\infty} c_n x^n$

$$(x-4) \sum_{n=2}^{\infty} c_n n(n-1) x^{n-2} - \sum_{n=1}^{\infty} c_n n x^{n-1} - 2 \sum_{n=0}^{\infty} c_n x^n = 0$$

$$-8c_2 - c_1 - 2c_0 + \sum_{k=1}^{\infty} [c_{k+1}(k+1)(k+2) - 4c_{k+2}(k+1)(k+2) - 2c_k] x^k = 0$$

$$\begin{cases} c_2 = -\frac{c_1 + 2c_0}{8} \\ c_{k+2} = \frac{(k+1)(k+2)c_{k+1} - 2c_k}{4(k+1)(k+2)}, k=1, 2, \dots \end{cases}$$

Case 1: $c_0 = 0, c_1 \neq 0$

$$c_2 = -\frac{c_1}{8}$$

$$c_3 = \frac{c_1}{12}, c_4 = 0$$

Case 2: $c_0 \neq 0, c_1 = 0$

$$c_2 = -\frac{c_0}{4}, c_3 = 0$$

$$c_4 = \frac{c_0}{96}$$

Thus,

$$y = c_1 \left(x - \frac{x^2}{8} - \frac{1}{12} x^3 + \dots \right)$$

$$y_1 = c_0 \left(1 - \frac{x^2}{4} + \frac{x^4}{96} + \dots \right)$$

$y = c_1 y_1 + c_0 y_2$ is the general solution

2) $p(x) = 3(x^4-1); q(x) = -1(1-x^3)$

$$r(r-1) - 3r - 1 = 0; r^2 - 4r - 1 = 0$$

$$r_1 = 2 - \sqrt{5}; r_2 = 2 + \sqrt{5}$$

3) $y = \sum_{n=0}^{\infty} c_n x^{n+\frac{1}{2}}, y' = \sum_{n=0}^{\infty} c_n (n+\frac{1}{2}) x^{n-\frac{1}{2}}$

$$y'' = \sum_{n=0}^{\infty} c_n (n+\frac{1}{2})(n-\frac{1}{2}) x^{n-\frac{3}{2}}$$

$$\sum_{n=1}^{\infty} 2c_n (n+\frac{1}{2})(n-\frac{1}{2}) x^{n-1} + \sum_{n=1}^{\infty} c_n (n+\frac{1}{2}) x^{n-1} + \sum_{n=0}^{\infty} 2c_n x^n = 0$$

$$\sum_{k=0}^{\infty} 2c_{k+1} (k+\frac{3}{2})(k+\frac{1}{2}) x^k + \sum_{k=0}^{\infty} c_{k+1} (k+\frac{3}{2}) x^k + \sum_{k=0}^{\infty} 2c_k x^k = 0$$

$$\Rightarrow c_{k+1} = \frac{-2c_k}{(k+1)(2k+3)}, k=0, 1, 2, \dots$$