Exercise 1.

(a) Verify that
\[ [\cosh(x)]^3 = \frac{1}{4} \cosh(3x) + \frac{3}{4} \cosh(x). \]

(b) Let \( L \) be a linear differential operator such that \( y_{p1} \) and \( y_{p2} \) are particular solutions of \( L(y) = \cosh(3x) \) and \( L(y) = \cosh(x) \), respectively. Find a particular solution of the DE: \( L(y) = [\cosh(x)]^3 \).
Exercise 2. Without solving the differential equation, verify that

\[ y = c_1 e^{2x} + c_2 e^{3x} + x^2 \]

is the general solution of the DE:

\[ y'' - 5y' + 6y = 6x^2 - 10x + 2. \]
Exercise 3. Show that the functions
\[ f_1 = e^x, \quad f_2 = xe^x, \quad f_3 = x^2 e^x \]
are linearly independent on \( I = \mathbb{R} \).
Exercise 4. Find a differential equation with general solution:

\[ y = c_1 e^{2x} + c_2 e^x \cos(x) + c_3 e^x \sin(x) + c_4 x e^x \cos(x) + c_5 x e^x \sin(x) + x^2, \]

where \(c_1, c_2, c_3, c_4\) and \(c_5\) are real parameters.
Exercise 5. Consider the following differential equation:

$$2t^2 y'' + 3ty' - y = 0.$$ 

Given that $y_1 = t^{-1}$ is a solution of the DE:

(a) find a suitable transformation to reduce the DE to a first order DE.

(b) find all solutions to the DE.