Exercise 1 (6 pts). Consider the relation $R = \{(x, y) \in \mathbb{N} \times \mathbb{N} \mid x + 4y \text{ is odd}\}$.

(1) Define the relation $R^{-1}$.

(2) Is $R = R^{-1}$?
Exercise 2 (6 pts). Let $R$ be an equivalence relation on a set $A$ and $B \subseteq A$. Set $S = R \cap (B \times B)$.

1. Show that $S$ is an equivalence relation on $B$.
2. Show that for all $x \in B$, $[x]_S = [x]_R \cap B$. 
Exercise 3 (10 pts). Let $a, b \in \mathbb{Z}$. Show that the relation $R$ defined on $\mathbb{Z}$ by:

$$aRb \quad \text{iff} \quad a^2 + b^2 \quad \text{is even}$$

is an equivalence relation and determine its distinct equivalence classes.
Exercise 4 (10 pts). Give the addition and multiplication tables on $\mathbb{Z}_9$.
Find all integers $x$ such that $6x \equiv 3 \pmod{9}$. 
Exercise 5 (8 pts). Let $g : \mathbb{Z} \rightarrow \mathbb{Z}$ be the function defined by: $g(x) = 4x + 1$.

(1) Determine $g(E)$, where $E$ is the set of all even integers. Is $11 \in g(E)$?

(2) Determine $g^{-1}(\mathbb{N})$, $g^{-1}(E)$ and $g^{-1}(O)$, where $O$ is the set of all odd integers.
Exercise 6 (10 pts). Let $f : A \to B$ be a function.

(a) Show that $f$ is injective if and only if $f^{-1}(f(C)) = C$, for all $C \subseteq A$.
(b) Show that $f$ is surjective if and only if $f(f^{-1}(D)) = D$, for all $D \subseteq B$. 
Exercise 7 (10 pts). Let $f : \mathbb{R} \setminus \{11\} \rightarrow \mathbb{R} \setminus \{7\}$, defined by $f(x) = \frac{7x + 1}{x - 11}$. Show that $f$ is a bijection and find $f^{-1}$. 
PART B
Exercise 8 (10 pts). Let \( n \geq 2 \) be an integer. Using the PHP (Pigeonhole Principle), prove that in a collection of \( n + 1 \) distinct integers, there are distinct integers \( x \) and \( y \) such that \( x - y \) is a multiple of \( n \).
Exercise 9 (10 pts). Find the number of positive integers $x$ between 1 and $10^4$ such that $7 \nmid x$ and $13 \nmid x$. 
Exercise 10 (10 pts). Let $A = \{1, 2, 3\}$. Find the distinct equivalence classes of the equivalence relation $\sim$ defined on $S = P(A)$ by:

$$E \sim F \text{ iff } |E| = |F|.$$
Exercise 11 (10 pts).

1. Give an explicit injection from \( \mathbb{N} \times \mathbb{N} \) into \( \mathbb{N} \).

2. For any integer \( k \geq 3 \) give an explicit injection from \( \mathbb{N} \times \mathbb{N} \times \mathbb{N} \ldots \times \mathbb{N} \) (\( k \) times) into \( \mathbb{N} \).

3. Deduce that \( \mathbb{N} \times \mathbb{N} \times \mathbb{N} \ldots \times \mathbb{N} \) and \( \mathbb{N} \) have the same cardinality.
Exercise 12 (10 pts). Let \( A = \{1, 2, 3, 4, 5, 6\} \) and \( B = \{a, b, c, d, e\} \).

1. Find all the equivalence relations on \( A \) with exactly 5 distinct equivalence classes.

2. Show that if \( f : A \to B \) is an onto map then \( \{f^{-1}(\{b\}) \mid b \in B\} \) is a partition of \( A \).

3. How many onto maps are there from \( A \) to \( B \)?
Exercise 13 (10 pts). Let $a < b$ be real numbers and $a < c < b$. Show that $]a, b[\setminus\{c\}$ and $\mathbb{R}$ have the same cardinality.