King Fahd University of Petroleum & Minerals  
Department of Mathematics & Statistics  
Math 301  Major Exam 1  
The Second Semester of 2015-2016 (152) 
Time Allowed: 120 Minutes

Name: ___________________________ ID#: ________________
Instructor: _______________________ Sec #: _______ Serial #: _______

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

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Q:1 (6+6 points) Given the position vector \( \vec{r}(t) = 2\sqrt{2} \, t \, \hat{i} + e^{2t} \, \hat{j} + e^{-2t} \, \hat{k} \) of a curve \( C \):

(a) Find \( \frac{d}{dt} \left[ \vec{r}(t) \times \vec{r}'(t) \right] \) at \( t = 0 \).

(b) Find the length of the curve for \( 0 \leq t \leq 2 \).
Q: 2 (8+6 points) Let \( f(x, y, z) = xy - 3y^2z + x^2z \).

(a) Find the directional derivative of \( f(x, y, z) \) at \((1, -1, 0)\) in the direction of \( \hat{i} + 2\hat{j} + 3\hat{k} \).

(b) Find the direction in which the function \( f(x, y, z) \) decreases most rapidly at \((1, 1, 1)\) and the value of minimum rate of change at this point.
Q:3 (6+8 points) Let \( g(x, y, z) = xyz \) and \( \vec{F} = y \hat{i} - x \hat{j} + z \hat{k} \). Calculate

(a) \( \nabla \cdot (g \vec{F}) \) at \((0, 1, 1)\)

(b) \( \nabla \times (g \vec{F}) \) at \((1, 1, 1)\).
Q:4 (15 points) Find work done by the force $\vec{F} = (y + yz \cos x) \hat{i} + (x + z \sin x) \hat{j} + y \sin x \hat{k}$ along the curve $\vec{r}(t) = 2t \hat{i} + (1 + \cos t) \hat{j} + 4 \sin t \hat{k}$ for $0 \leq t \leq \frac{\pi}{2}$.
Q:5 (15 points) Use Green’s theorem to evaluate the line integral
\[ \oint_C (-16y + \sin x^2)dx + (4e^y + 3x^2)dy, \]
where \( C \) is the positively oriented boundary of the region bounded by the graphs of \( y = x, y = -x, \) and \( x^2 + y^2 = 4 \) with \( y \geq 0. \)
Q:6 (15 points) Use Stokes’ theorem to evaluate the integral $\int_{C} \vec{F} \cdot d\vec{r}$ where $\vec{F} = y^3\hat{i} - x^3\hat{j} + z^3\hat{k}$ and $C$ is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $x + y + z = 1$.
Q:7 (15 points) Use divergence theorem to evaluate \( \iiint_D \mathbf{F} \cdot \mathbf{n} \, dS \)

where \( \mathbf{F} = z^2 \sin y \mathbf{i} + 5x^2z \mathbf{j} + 3z^2 \mathbf{k} \) and \( D \) the region bounded by the surface \( S \) given by

\[
z = \sqrt{9 - x^2 - y^2}, \quad x^2 + y^2 = 4, \quad z = 0.
\]