King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

MATH 302, Semester 152 (2015-2016)

FINAL EXAM
Thursday May 19, 2016

Allowed Time: 3 Hours

Student Name:
Student ID Number:
Section Number:
Instructor’s Name:

Instructions:
1. Write neatly and legibly -- you may lose points for messy work.
2. Show all your work -- no points for answers without justification.
3. Calculators and mobiles are not allowed - do not bring them in to the exam room.
4. Make sure that you have 10 pages (9 questions + 1 cover page).

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<th>Problem No.</th>
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Coordinator: Dr Nadeem A. Malik
Q1.

(a) Find all the points where $f'(z)$ exists where $f(z) = x^3 + i(1 - y)^3$ [4 pts]

(b) Find the residue of the function $f(z) = \sin(2/z)$ at $z = 0$ [4 pts]

(c) Find the residue of the function $f(z) = z/(z^2 + 1)^2$ at $z = -i$ [4 pts]
Q2. Find the 3 values of $w$ or which, $w^3 = 1 - \sqrt{3}i$. Sketch the solutions. [9 pts]
Q3. Find all complex numbers satisfying, $\sin z = \cosh 4$. [9 pts]
Q4. Evaluate \( \int_C \frac{dz}{z-1} \) along the contour C shown below. [15 pts]
Q5. Expand the function $f(z) = \frac{z^2}{(z-1)(z-2)}$ as a Laurent series valid for, $1 < |z| < 2$. 

[15 pts]
Q6. Use Cauchy’s integral formula to evaluate,

\[ \oint_C \left[ \left( \frac{2z}{z-1} \right)^3 - \left( \frac{z-1}{2z} \right)^3 \right] \, dz. \]

C is the circle \(|z| = 2\). [15 points]
Q7. Evaluate the integral,

$$\oint_C \left( \frac{\cos z}{z^3 + z^2} + \frac{e^{2z}}{z^4 + \pi^4} \right) dz$$

$C$ is the circle $|z| = 2$. [20 points]
Q8. Using complex integral methods, evaluate the integral,

\[ \int_{0}^{2\pi} \frac{\cos 2\theta \ d\theta}{(5 - 4 \cos \theta)} \]

[18 points]
Q9. Evaluate the Cauchy Principle Value (P.V.) of,

\[ \int_{-\infty}^{\infty} \frac{x \cos x \, dx}{(x^3 - x^2 + x - 1)} \]

[22 points]