King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 430 Exam 02
The Second Semester of 2015-2016 (152)
Time Allowed: 90 Minutes

Name: ____________________________  ID#: __________________
Section/Instructor: ________________  Serial #: __________________

• Mobiles and calculators are not allowed in this exam.
• Provide all necessary steps required in the solution.

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Q1: (8 + 2 points) Let \( z = x + iy \) and \( f(z) = 4x + i5y \).

(a) Show that \( f(z) \) is differentiable only \( x- \) axis and a vertical line \( x = b \) for some real number \( b \).

(b) Show that \( f(z) \) is nowhere analytic.

Q2: (6 + 2 + 6 points)(a) State the Cauchy-Riemann equations. Show that 
\[ f(z) = e^{x^2-y^2} \left[ \cos(2xy) + i \sin(2xy) \right] \]
is entire and find its derivative.

(b) Verify that the function \( u(x, y) = x^3 - 3xy^2 - 5y \) is harmonic in the entire complex plane.

(c) Find a harmonic conjugate of \( u(x, y) = x^3 - 3xy^2 - 5y \).

Q3: (6 + 2 + 2 points) (a) Express \( P_5(z) = z^5 - 1 \) as a product of linear and quadratic factors.

(b) Prove that \( \cos(z) = 0 \) iff \( z = \frac{\pi}{2} + k\pi, \ k \in I \).

(c) Find all numbers \( z \) (if any) such that \( e^{iz} = 3 \).

Q4: (4 + 3 + 4 + 3 points) (a) Determine the domain of analyticity for \( f(z) = \log(4 + i - z) \).

(b) Find all the values of \( (1 + i)^3 \).

(c) Derive the identity [write every step line by line]
\[ \sin^{-1}z = -i \log[iz + (1 - z^2)^{1/2}] \]

(d) Find all values of \( z \) satisfying \( \sin(z) = 2 \).