Name:__________________________ ID #:__________________________

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Q.No.1: (1+2+2+2+1+2 = 10 points) Find the number of possible 5 character passwords under the following restrictions: (Note there are 26 letters in the alphabet.)

(a) All characters must be lower case letters.

(b) All characters must be lower case letters and distinct.

(c) Letters and digits must alternate and be distinct (as in “1w2x9” or “a1b2c”).

(d) All characters must be lower case, distinct, and in alphabetical order. (e.g., “abfgh” is allowed, but not “bafgh”).

(e) The password can only contain the upper case letters A and B.
(f) The word can only contain the upper case letters A and B, and must contain each of these letters.

Q.No.2:- (2+3 = 5 points) How many ways are there to seat 10 people, consisting of 5 couples, in a row of seats (10 seats wide) if
(a) the seats are assigned at random?

(b) all couples are to get adjacent seats?

Q.No.3:- (6 points) Six people get into an elevator at the ground floor of a hotel which has 10 upper floors. Assuming each person gets off at a randomly chosen floor, what is the probability that no two people get off at the same floor?
Q.No.4: (3+3+2 = 8 points) Assume a committee of 10 has to be selected from a group of 100 people, of which 40 are men and 60 are women.

(a) How many ways are there to choose such a committee?

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(b) How many ways are there to choose the committee so that exactly half of the members are men?

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(c) What is the probability that a randomly selected committee of 10 consists of exactly 5 men and 5 women?

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Q.No.5: (5+5+5 = 15 points) A box contains $n$ white and $m$ black balls, where $n$ and $m$ are positive numbers.

(a) If two ball are randomly withdrawn, without replacement, what is the probability that they are the same color?
(b) If a ball is randomly withdrawn and then replaced before the second one is drawn, what is the probability that the withdrawn balls are the same color?

(c) Show that the probability in part (b) is always larger than the one in part (a) if \( m \neq n \).
Q.No.6: (6 points) Prove that \( P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A^c \cap B \cap C) - P(A \cap B^c \cap C) - P(A \cap B \cap C^c) - 2P(A \cap B \cap C) \)

Hint: \( P(E \cap F^c) = P(E) - P(E \cap F) \)