Problem 1 (40 pts)

Given the following linear program (P):

\[
\begin{align*}
\max_{x_1, x_2} & \quad x_1 + 3x_2 \\
\text{s.t.} & \quad x_1 + 2x_2 \leq 4, \\
& \quad 3x_1 + x_2 \leq 4, \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]

(a) Solve the linear program (P) using the Simplex algorithm.  
(20 points)
(b) Write the dual program \((D)\) corresponding to \((P)\).
(5 points)

(c) Solve the dual linear program \((D)\) using the Dual Simplex algorithm.
(15 points)
Problem 3 (30 Points)

The following Simplex tableau corresponds to the representation of a basic feasible solution of a linear program during its optimization.

<table>
<thead>
<tr>
<th>$c^t$</th>
<th>?</th>
<th>?</th>
<th>?</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Basis</strong></td>
<td>$x_1$</td>
<td>$x_2$</td>
<td>$x_3$</td>
<td>$e_1$</td>
<td>$e_2$</td>
<td>$e_3$</td>
</tr>
<tr>
<td>$x_1$</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$e_2$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td><strong>RC</strong></td>
<td>0</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

(a) Complete the missing values in the Tableau.(10 points)
(b) Perform a single pivot iteration in case the objective has to be maximized.
(5 points)
(c) Perform a single pivot iteration in case the objective has to be minimized. 
(5 points)

(d) Give a possible original expression of the linear program. (10 points)
**Problem 3 (30 pts)**

Consider the following linear program:

\[
\begin{align*}
\text{max} & \quad 5x_1 + 3x_2 + 2x_3 \\
\text{s.t.} & \quad 2x_1 + x_2 + x_3 \leq 4, \\
& \quad x_1 + 2x_2 + x_3 \geq 5, \\
& \quad x_1, x_2, x_3 \geq 0.
\end{align*}
\]

Solve the linear program using the Revised Simplex algorithm. (30 points)

N.B.: You would be graded on 20 if you solve using the Simplex algorithm.