King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
2015-2016 (Term 153)
Introduction to Differential Equations & Linear Algebra
(MATH 260)
Exam II

Student Name:  
ID #:  

Section #:  
Serial #:  

Instructions
1. Justify your answers. No credit is given for (correct) answers not supported by work.
2. Write clearly. Marks may be deducted for messy work.

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1. [14pts] Use the cofactor matrix of $A$ to find $A^{-1}$ if $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & 0 & -2 \\ 2 & -1 & 0 \end{bmatrix}$. 
2. [10pts] Let \[
\begin{vmatrix}
  a & b & c \\
  d & e & f \\
  g & h & j
\end{vmatrix}
= 4.
\] Use elementary operations to find the value of the determinant

\[
\begin{vmatrix}
  2a - b & 3c & b \\
  2d - e & 3f & e \\
  2g - h & 3j & h
\end{vmatrix}
\]
3. [8pts] Let $S$ consist of all vectors $(x, y, z, w)$ of $\mathbb{R}^4$ such that $xy + z + w = 0$. Is $S$ a subspace of $\mathbb{R}^4$? Justify.
4. [8pts] Find all possible values of $k$ for which the vectors $(1, 2, 4), (1, 3, 9), (1, k, k^2)$ of $\mathbb{R}^3$ are linearly independent.
Given that \( y_1 = \cos 2x \) is a solution of the DE

\[
y^{(4)} - y^{(3)} + 2y'' - 4y' - 8y = 0,
\]

find:

(a) a second solution \( y_2 \) of the DE such that \( y_1 \) and \( y_2 \) are linearly independent.

(b) the general solution of the DE.
6. [8pts] Are the functions \( f(x) = x, g(x) = e^x - 1, h(x) = xe^x \) linearly independent on \((-\infty, \infty)\)? Justify.
7. [14pts] Solve the IVP: \[ y''' - 3y'' + 4y' - 2y = 0, \quad y(0) = 2, \quad y'(0) = 3, \quad y''(0) = 4. \]
8. [14pts] Find the form of a particular solution of the DE:

\[(D - 1)^2 (D^2 - 9) y = e^{3x} + 2e^{-3x} + 3xe^x.\]