King Fahd University of Petroleum and Minerals
MATH-302
Quiz 4

Name:- **ID:-** **Sec.: 01**

(1) (a) Find an orthogonal matrix **P** that diagonalizes $\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

(b) Find $\mathbf{P}^{-1}$.

(c) Find $\mathbf{P}^{-1}\mathbf{AP}$.

(d) Find the eigenvalues of A^{-1}.

(a) $$|A - \lambda I| = \begin{vmatrix} -\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = -(\lambda+1)(\lambda+\sqrt{2}-1)(\lambda-\sqrt{2}-1)$$

$$\lambda = -1 \quad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow K_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda_2 = \sqrt{2}+1 \quad \begin{pmatrix} -\sqrt{2}-1 & 1 & 1 \\ 1 & -\sqrt{2} & 1 \\ 1 & 1 & -\sqrt{2}-1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} \quad K_2 = \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$$

$$\lambda_3 = -\sqrt{2}+1 \quad \begin{pmatrix} \sqrt{2}-1 & 1 & 1 \\ 1 & \sqrt{2} & 1 \\ 1 & 1 & \sqrt{2}-1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} \quad K_3 = \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$$

$K_i \cdot K_j = 0$ for all i, j. K_1, K_2, K_3 are mutually orthogonal.

$$P = \begin{pmatrix} -1/\sqrt{2} & 1/2 & 1/2 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 1/\sqrt{2} & 1/2 & 1/2 \end{pmatrix}$$

(b) $$P^{-1} = P^T = \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/2 & \sqrt{2}/2 & 1/2 \\ 1/2 & -\sqrt{2}/2 & 1/2 \end{pmatrix}$$

(c) $$P^{-1}AP = D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \sqrt{2}+1 & 0 \\ 0 & 0 & -\sqrt{2}+1 \end{pmatrix}$$

(d) The eigenvalues of A^{-1} are $\frac{1}{\lambda_i} = -1, \frac{1}{\sqrt{2}+1}, \frac{1}{-\sqrt{2}+1}$.

(2) Use Gauss-Jordan method to find the inverse of $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{pmatrix}$.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 9 & 3 & 0 & 1 & 0 \\ 1 & 0 & 4 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 5 & -3 & -2 & 1 & 0 \\ 0 & -2 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[R_2 + \frac{5}{2}R_3]{R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & 0 & 1 \\ 0 & 0 & -1/2 & -9/2 & 1 & 5/2 \\ 0 & -2 & 1 & -1 & 0 & 1 \end{array}\right) \xrightarrow[R_{23}]{-\frac{1}{2}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & 0 & 1 \\ 0 & 1 & -1/2 & 1/2 & 0 & -1/2 \\ 0 & 0 & -1/2 & -9/2 & 1 & 5/2 \end{array}\right)$$

$$\xrightarrow{-2R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & 0 & 1 \\ 0 & 1 & -1/2 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1 & 9 & -2 & -5 \end{array}\right) \xrightarrow[R_1 - 4R_3]{R_2 + \frac{1}{2}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -36 & 8 & 21 \\ 0 & 1 & 0 & 5 & -1 & -3 \\ 0 & 0 & 1 & 9 & -2 & -5 \end{array}\right)$$