KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

DEPARTMENT OF MATHEMATICS & STATISTICS

AS 498 - Term 161

Date: December 14, 2016   **Exam 2**   **Duration: 120 minutes**

Name:

ID #: _ _ _ _ _ _ _ _ _   **Section #:** __01__   **Serial #:** _ _

Instructions:

- Write neatly and eligibly. You may lose points for messy work.
- Show all your work. No points for answers without justification.
- ***Electronic approved calculators*** are allowed. Mobiles are NOT allowed.
- Make sure that you have **7** pages of problems (Total of **5** Questions).

Question #	Score
1	/ 10
2	/ 10
3	/ 15
4	/ 20
5	/ 25
Total	**/ 80**

1. Suppose $X(t)$ is an ***Ornstein-Uhlenbeck*** process defined by:
$$dX(t) = 2[4 - X(t)]dt + 8dZ(t)$$
where $Z(t)$ is a standard Brownian motion. If $Y(t) = 1/X(t)$ and:
$$dY(t) = a\big(Y(t)\big)dt + b\big(Y(t)\big)dZ(t)$$
determine $a(0.5)$.   **(10 points)**

2. Consider the Black-Scholes framework. For a stock and a derivative security on the stock, you are given:

   (i)   The continuously compound risk-free interest rate is $r = 5.5\%$.
   (ii) The time-t price of the stock is $S(t)$.
   (iii) The time-t price of the derivative security is $V(S(t), t) = e^{rt} \ln[S(t)]$.
   (iv) The stock's volatility is 30%.
   (v)   The stock pays dividends continuously at a rate proportional to its price.
   (vi) The derivative security does not pay dividends.

   Find δ, the dividend yield on the stock.   **(10 points)**

   Hint: Find the three Greek letters (Δ, Γ and θ), then apply the Black-Scholes Equation.

3. For binomial tree model for a stock, you are given:

    (i)   The current stock price is 40.
    (ii) In each half year, the price will either rise by 20% or fall by 20%.
    (iii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 3%.
    (iv) The continuously compounded risk-free interest rate is 5%.

    Approximate the time-0 delta, gamma and theta of a 1-year at the money European call option on the stock. **(15 points)**

4. Let $S(t)$ be the time-t price of nondividend paying stock. You are given:

   (i) $S(t)$ follows a geometric Brownian motion with $S(0) = 30$.
   (ii) The stock volatility is 20%.
   (iii) $P(S(2) < 20) = 2.5\%$.

   Find the following:

   (a) The expected rate of appreciation on the stock.

   (b) $E(S(2) \mid S(2) < 20)$

   (c) A 95% lognormal prediction interval for $S(4)$.

(d) The probability that a European call option with a strike price of 35 and a maturity date in three months will be exercised.

(e) The smallest integer-valued strike price, K, for which an investor will exercise a European put option in three months with probability 99%.

(3 + 4 + 4 + 4 + 5 = 20 points)

5. Consider a stock that follows a geometric Brownian motion. You are given:

   (i)   The time-t price of the stock is $S(t)$.
   (ii) The current stock price is 50.
   (iii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 1%.
   (iv) The expected rate of appreciation on the stock is 4%.
   (v)   The sharp ratio is 0.20.
   (vi) The continuously compound risk free interest is 3%.

   Find the following:

   (a) The stock's volatility.

   (b) The prepaid forward price for a claim that pays $S^{-1}(0.5)$ after 6 months.

(c)   The price of a 3-month 45-strike cash-or-noting put option on the stock.

(d)   The price of a 3-month 55-strike European call option on the stock.

(e)   The price of a 3-month 55-strike European put option on the stock.

(f)   The price of a straddle that pays $|S(0.25) - 55|$ after 3 months.

(4 + 5 + 4 + 5 + 4 + 3 = 25 points)