

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DEPARTMENT OF MATHEMATICS & STATISTICS

AS498 - Section 01 (Term 161)

Date: January 05, 2017

Quiz 6

Duration: 30 minutes

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Family Name: \_\_\_\_\_ ID #: \_\_\_\_\_ Serial #: \_\_\_\_\_

1. Let  $V(r, t, T)$  be the time- $t$  price of an interest rate derivative that matures at time  $T$ , when the time- $t$  short rate is  $r$ . You are given:

(i) The stochastic process of  $V$  is given by:

$$dV(r, t, T) = \alpha(r, t, T)V(r, t, T)dt - q(r, t, T)V(r, t, T)dZ(t)$$

where  $Z(t)$  is a standard Brownian motion.

(ii) The Sharp ratio of the interest rate risk is of the form  $\phi(r, t) = kt\sqrt{r}$ , where  $k$  is a constant.

(iii)  $\alpha(9\%, 1, 2.5) = 0.33$  and  $q(9\%, 1, 2.5) = 0.5$ .

(iv)  $\alpha(9\%, 2, 2.5) = 0.04$ .

Find the following:

(a) The value of the constant  $k$ .

(b)  $q(9\%, 2, 2.5)$ .

(4 + 4 = 8 points)

2. Let  $r(t)$  be the time- $t$  short rate and let  $P(r, t, T)$  be the time- $t$  price of a zero-coupon bond that matures at time  $T$ , when the time- $t$  short rate is  $r(t)$ . You are given:

(i) The stochastic process of  $r(t)$  is given by:

$$dr(t) = 0.25[b - r(t)]dt + 0.2dZ(t)$$

where  $b$  is a positive constant and  $Z(t)$  is a standard Brownian motion.

(ii) The Sharp ratio of the interest rate risk is 10%.

(iii) The stochastic process of  $P$  is given by:

$$dP(r, t, T) = \alpha(r, t, T)P(r, t, T)dt - q(r, t, T)P(r, t, T)dZ(t)$$

(iv)  $\lim_{T \rightarrow \infty} \frac{-1}{T} \ln P(5\%, 0, T) = 6\%$ .

Find the following:

(a)  $q(5\%, 2, 4)$ .

(b)  $\alpha(5\%, 2, 4)$ .

(c) The value of the constant  $b$ .

**(3 + 4 + 5 = 12 points)**